Dear Dr. Arora,

On behalf of my co-authors, I would like to resubmit the revised manuscript “esd-2018-71” titled "Investigating the Applicability of Emergent Constraints” for publication in Earth System Dynamics. All referee comments are addressed comprehensively with additional analyses and revisions to the manuscripts, as described in the attached point-by-point response files.

Please find below the revised manuscript in marked-up version with revisions highlighted in red and blue.

Thank you for your consideration.

Yours sincerely,

Alexander Winkler (for all co-authors)
Investigating the Applicability of Emergent Constraints on Multi-Model Projections: Case Study of Constraining Vegetation Productivity With Observed Greening Sensitivity

Alexander J. Winkler\textsuperscript{1,2}, Ranga B. Myneni\textsuperscript{3}, and Victor Brovkin\textsuperscript{1}

\textsuperscript{1}Max Planck Institute for Meteorology, Bundesstrasse 53, 20146 Hamburg, Germany
\textsuperscript{2}International Max Planck Research School on Earth System Modelling, Bundesstrasse 53, 20146 Hamburg, Germany
\textsuperscript{3}Department of Earth and Environment, Boston University, Boston, Massachusetts 02215, USA

Correspondence: Alexander J. Winkler (alexander.winkler@mpimet.mpg.de)

Abstract. Recent research on Emergent Constraints (EC) has delivered promising results in narrowing down uncertainty in climate predictions. The method utilizes a measurable variable (predictor) from the recent historical past to obtain a constrained estimate of change in a difficult-to-measure variable an entity of interest (predictand) at a potential future \textit{CO}_2 concentration (forcing) from multi-model projections. This procedure critically depends on, first, accurate estimation of the predictor from observations and models, and second, on a robust relationship between inter-model variations in the predictor-predictand space. We Here, we investigate issues related to these two themes in this article, using a carbon cycle case study using observed vegetation greening sensitivity to forcing during the satellite era \textit{CO}_2 forcing as a predictor of change in photosynthesis (Gross Primary Productivity [GPP]) of the Northern High Latitudes region (60° N–90° N, NHL, GPP) for a doubling of pre-industrial concentration in the atmosphere \textit{CO}_2 concentration. Greening sensitivity is defined as changes in annual maximum of green leaf area index (LAI\textsubscript{max}) per unit \textit{CO}_2 forcing realized through its radiative and fertilization effects. We first address the question of how to realistically characterize the greening sensitivity-predictor of a large area, the NHL, (e.g., greening sensitivity in the northern high latitudes region) from pixel-level LAI\textsubscript{max} data. This requires an investigation into uncertainties in LAI\textsubscript{max} the observational data source and an evaluation of the spatial and temporal variability in greening sensitivity to forcing the predictor in both the data and model simulations. Second, the relationship between greening sensitivity and ΔGPP-predictor-predictand relationship across the model ensemble depends on a strong coupling among between the two variables, i.e., simultaneous changes in GPP and LAI\textsubscript{max}. This coupling depends in a complex manner on the magnitude (level), time-rate of application (scenarios) and effects (radiative and/or fertilization) of \textit{CO}_2 forcing. We investigate how each one of these three aspects of forcing can impair the EC estimate of the predictand (ΔGPP). Accounting for uncertainties in greening sensitivity and stability of the relation between inter-model variations results in a quantitative estimate of Our results show that uncertainties in the uncertainty (±0.2 Pg C yr\textsuperscript{-1}) on constrained GPP enhancement (ΔGPP = +3.4 Pg C yr\textsuperscript{-1}) for a doubling of pre-industrial atmospheric concentration in NHL. This ΔGPP is 60% larger than the conventionally used average of model projections. The illustrated EC method can primarily originate from a lack of predictor comparability between models and observations, temporal variability, and the observational data source of the predictor. The disagreement between models on the mechanistic
behavior of the system under intensifying forcing limits the EC applicability. The here illustrated limitations and sources of uncertainty in the EC method go beyond carbon cycle research and are generally relevant for Earth system sciences.
1 Introduction

Earth system models (ESMs) are powerful tools to predict responses to a variety of forcings such as increasing atmospheric concentration of greenhouse gases and other agents of radiative forcing (Klein and Hall, 2015). Still, longterm ESM projections of climate change can have substantial uncertainties. This can be due to poorly understood processes in some cases, and in others, to missing or simplified representations called parameterizations (Flato et al., 2013; Klein and Hall, 2015; Knutti et al., 2017). Certain important processes, especially in the atmosphere, happen at spatial scales finer than can be possibly represented in current ESMs. Consequently, certain key aspects of the system, such as variability, various phenomena in the system ranging from local extreme precipitation events and to large-scale climate modes, can be poorly simulated (Flato et al., 2013).

Errors propagate and can be amplified through feedbacks among interacting components in the Earth system, resulting in biases whose origins can be difficult to identify (Flato et al., 2013). Furthermore, an inherent component of the Earth climatic system, its internal natural variability, is complicated to represent and simulate in models (Flato et al., 2013; Klein and Hall, 2015).

Model Intercomparison Projects aim is to explore these uncertainties by coordinating a wide range of simulation setups focusing on internal variability, boundary conditions, parameterizations, etc. (Taylor et al., 2012; Eyring et al., 2016; Flato et al., 2013; Knutti et al., 2017). Models developed at various institutions are driven with the same forcing information (e.g. historical forcing) or with identical idealized boundary conditions. However, each modeling group decides which of the processes to consider and implement in their ESM. The conventional approach of handling these multi-model ensembles is to use unweighted ensemble averages (Knutti, 2010; Knutti et al., 2017). This assumes that the models are independent of one another and equally good at simulating the climate system (Flato et al., 2013; Knutti et al., 2017).

The large spread between model projections suggests that this assumption is not valid. Therefore, alternate methods have been developed to extract results more accurate than multi-model averages (e.g., model weighting scheme based on performance and interdependence, Knutti et al., 2017). The concept of Emergent Constraints arises in this context, namely, as a method to reduce uncertainty in ESM projections relying on historical simulations and observations (Hall and Qu, 2006; Boé et al., 2009; Cox et al., 2013; Klein and Hall, 2015; Cox et al., 2018).

The two key parts of an Emergent Constraint (EC) based method are a linear relationship arising from the collective behavior of a multi-model ensemble and an observational estimate for imposing the said constraint (Fig. 1). The linear relationship is a physically (or physiologically) based correlation between inter-model variations in an observable entity of the contemporary climate system (predictor) and a projected variable (predictand) that is usually difficult to observe or not observable at all. Combining the emergent linear relationship with observations of the predictor sets a constraint on the predictand (Knutti et al., 2017; Klein and Hall, 2015; Cox et al., 2013; Flato et al., 2013)(Cox et al., 2013; Flato et al., 2013; Klein and Hall, 2015; Knutti et al., 2017). Many such ECs have been identified and reported, as briefly summarized below.
Hall and Qu (2006) proposed a constraint on projections of snow-albedo feedback based on the correlation between large inter-model variations in feedback strength of the current seasonal cycle. The EC was first established for the CMIP3 ensemble and confirmed for phase five of the Coupled Model Intercomparison Project (CMIP5) (Qu and Hall, 2014; Flato et al., 2013) (CMIP5; Flato et al., 2013; Qu and Hall, 2014). Several EC studies followed with the goal of reducing uncertainty in projections of the cloud feedback under global warming, as reviewed by Klein and Hall (2015) and Klein and Hall (2015). It is thought that erroneous representation of low-cloud feedback in ESMs contributes essentially to the large uncertainty in equilibrium climate sensitivity (ECS, 1.5 to 5 K), i.e. warming for a doubling of pre-industrial atmospheric concentration (2×CO₂) (Klein and Hall, 2015; Sherwood et al., 2014). CO₂ concentration (2×CO₂; Sherwood et al., 2014; Klein and Hall, 2015). Recently, Cox et al. (2018) presented a different approach to constrain ECS based on its relationship to variability of global temperatures during the recent historical warming period. They reported a constrained ECS estimate of 2.8 K for 2×CO₂ (66% confidence limits of 2.2 – 3.4 K).

The concept of EC also found its way into the field of carbon cycle projections. A series of studies analyzed the extent to which inter-annual atmospheric CO₂ variability can serve as a predictor of longterm temperature sensitivity of terrestrial tropical carbon storage. Cox et al. (2013) and Wenzel et al. (2014) reported an emergent linear relationship, although with different slopes for CMIP3 and CMIP5 ensembles, resulting in slightly divergent constrained estimates (CMIP3: -53 ± 17 Pg C K⁻¹, CMIP5: -44 ± 14). Wang et al. (2014) however were unable to detect a similar relationship between the proposed predictor and predictand. Recently, Lian et al. (2018) presented an EC estimate of the global ratio of transpiration to total terrestrial evapotranspiration (T/ET), which is substantially higher (0.62 ± 0.06) than the unconstrained value (0.41 ± 0.11). For the marine tropical carbon cycle, Kwiatkowski et al. (2017) identified an emergent relationship between the longterm sensitivity of tropical ocean net primary production (NPP) to rising sea surface temperature (SST) in the equatorial zone and the interannual sensitivity of NPP to El Niño/Southern Oscillation driven SST anomalies. Tropical NPP is projected to decrease by 3 ± 1% for 1 K increase in equatorial SST according to the observational constraint.

Similar results were reported for modeled extra-tropical terrestrial carbon fixation in a 2×CO₂ world. Plant productivity is expected to increase due to the fertilizing and radiative effects of rising atmospheric CO₂ concentration. Wenzel et al. (2016) focused on constraining the CO₂ fertilization effect on plant productivity in the northern high latitudes (60° N – 90° N, NHL) and the entire extra-tropical area in the northern hemisphere (30° N – 90° N) using the seasonal amplitude of longterm CO₂ measurements at different latitudes. They presented a linear relationship between the sensitivity of CO₂ amplitude to rising atmospheric CO₂ concentration and the relative increase in zonally averaged gross primary production (GPP) for 2×CO₂. The observed CO₂ amplitude sensitivities at respective stations provided a constraint on the strength of the CO₂ fertilization effect, namely 37% ± 9% and 32% ± 9% for the NHL and the extra-tropical region, respectively.
Focusing on the NHL, Winkler et al. (2019) investigated how both effects of CO$_2$ enhance plant productivity while assessing the feasibility of vegetation greenness changes as a constraint (Fig. 1). Enhanced GPP due to the physiological effect and ensuing climate warming is indirectly evident in large-scale increase in summer time green leaf area (Myneni et al., 1997a; Zhu et al., 2016). Historical CMIP5 simulations show that the maximum annual leaf area index (LAI$_{\text{max}}$, leaf area per ground area) increases linearly with both concentration and growing degree days (above 0°C, GDD$_0$) CO$_2$ concentration and temperature in NHL. To avoid redundancy from co-linearity between the two driver variables, but retain their underlying time trend and interannual variability, the dominant mode from a principal component analysis of and GDD$_0$ was used as the proxy driver (denoted $\omega$). This greening sensitivity (i.e., $\frac{\Delta \text{LAI}_{\text{max}}}{\Delta \omega}$) can be inferred for the overlapping historical period from simulations and observations alike. In all ESMs, changes in these changes in LAI$_{\text{max}}$ strongly correlate to changes in GPP arising from the combined radiative and physiological effects of enrichment strongly correlate with changes in LAI$_{\text{max}}$ in the historical simulations CO$_2$ enrichment. Thus, the large variation in modelled historical LAI$_{\text{max}}$ sensitivities-responses to the effects of CO$_2$ linearly maps to variation inΔGPP at 2×. Hence, this CO$_2$ in the CMIP5 ensemble. Thus linear relationship in inter-model variation between ΔGPP at 2×and historical greening sensitivities allows using the observed sensitivity-variations enables the usage of the observed longterm change in LAI$_{\text{max}}$ as an EC on ΔGPP at 2×in NHL (3.4 ± 0.2; Winkler et al., 2018). CO$_2$ in NHL (3.4 ± 0.2 Pg C yr$^{-1}$; Winkler et al., 2019).

The EC method (Fig. 1) has its limitations. robustness of these EC estimates is debated, mainly because the EC approach is susceptible to methodological inconsistencies. For example, Cox et al. (2013), Wang et al. (2014) and Wenzel et al. (2015). Cox et al. (2013), Wang et al. (2014) and Wenzel et al. (2015) investigated on constraining future terrestrial tropical carbon storage using the same set of models and data. However, they arrived at different EC estimates and divergent conclusions. Some reasons for the failure and essential criteria required for successful application of the EC approach were described previously (Bracegirdle and Stephenson, 2012b; Klein and Hall, 2015), but this list is far from complete. The main focus thus far has been on caveats establishing an emergent linear relationship in a multi-model ensemble. However, large uncertainty on the constraint could result potentially from how the observational predictor is derived and compared to the modeled estimates. To account for this gap in the literature, a detailed investigation and description of the EC method in terms of its potential sources of uncertainty and the range of applicability are needed.

Here, we revisit the study of Winkler et al. (2019) and elaborate on key issues concerning sources of uncertainty regarding the constraint and applicability the robustness of the EC method.

Uncertainty on Uncertainty of the constrained estimate depends on (a) observed predictor and (b) modeled relationship, aside from the goodness-of-fit of the latter (green shading in Fig. 1). As for (a), the source of observations is an obvious first line of inquiry (Sect. 3.1). Spatial aggregation of data and model simulations introduces uncertainties, as the EC method is applied on large areal values of predictor and predictand. This is the subject of Sect. 3.2. The observed and modeled predictors are from the historical period. The representativeness, duration and match between data and models all introduce an uncertainty related to variations in the temporal domain – these are explored in (Sect. 3.3). The yellow shading in Fig. 1 represents the
total uncertainty on observed predictor from these three fronts. Regarding (b), the modeled linear relation varies (grey shading in Fig. 1) depending on three attributes of the forcing, i.e. CO₂ concentration change, its magnitude, rate and effect (Sect. 3.4 and 3.5). Lessons learned from analyses along these lines are presented in the conclusion section at the end.
2 Data and Methods

2.1 Observational data sets

2.1.1 Remotely sensed leaf area index

We used the recently updated version (V1) of the leaf area index data set (LAI3g) developed by (Zhu et al., 2013). It was generated using an artificial neural network (ANN) and the latest version (third generation) of the Global Inventory Modeling and Mapping Studies group (GIMMS) Advanced Very High Resolution Radiometer (AVHRR) normalized difference vegetation index (NDVI) data (NDVI3g). The latter have been corrected for sensor degradation, inter-sensor differences, cloud cover, observational geometry effects due to satellite drift, Rayleigh scattering and stratospheric volcanic aerosols (Pinzon and Tucker, 2014). This data set provides global and year-round LAI observations at 15-day (bi-monthly) temporal resolution and 1/12 degree spatial resolution from July 1981 to December 2016. Currently, this is the only available record of such length.

The quality of previous version (V0) of LAI3g data set was evaluated through direct comparisons with ground measurements of LAI and indirectly with other satellite-data based LAI products, and also through statistical analysis with climatic variables, such as temperature and precipitation variability (Zhu et al., 2013). The LAI3gV0 data set (and related fraction vegetation-absorbed photosynthetically active radiation data set) has been widely used in various studies (Anav et al., 2013; Forkel et al., 2016; Zhu et al., 2016; Mao et al., 2016; Mahowald et al., 2016; Piao et al., 2014; Poulter et al., 2014; Keenan et al., 2016). The new version, LAI3gV1, used in our study is an update of that earlier version.

We also utilized a more reliable but shorter data set from the Moderate Resolution Imaging Spectroradiometer (MODIS) aboard the NASA’s Terra satellite (Yan et al., 2016a, b). These data are well calibrated, cloud-screened and corrected for atmospheric effects, especially tropospheric aerosols. The sensor-platform is regularly adjusted to maintain a precise orbit. All algorithms, including the LAI algorithm, are physics-based, well-tested and currently producing sixth generation data sets. The data set provides global and year-round LAI observations at 16-day (bi-monthly) temporal resolution and 0.05° spatial resolution from 2000 to 2016.

Leaf area index is defined as the one-sided green leaf area per unit ground area in broadleaf canopies and as one-half the green needle surface area in needleleaf canopies in both observational and CMIP5 simulation data sets. It is expressed in units of m² green leaf area per m² ground area. Leaf area changes can be represented either by changes in annual maximum LAI ($LAI_{max}$) (Cook and Pau, 2013) or growing season average LAI. In this study, we use the former because of its ease and unambiguity, as the latter requires quantifying the start- and end-dates of the growing season, something that is difficult to do accurately in NHL (Park et al., 2016) with the low resolution model data. Further, $LAI_{max}$ is
less influenced by cloudiness and noise; accordingly, it is most useful in investigations of long-term greening and browning trends. The drawback of LAI\textsubscript{max}, is the saturation effect at high LAI values (Myneni et al., 2002). However, this is less of a problem in high latitudinal ecosystems which are less-densely vegetated, with LAI\textsubscript{max}, values typically in the range of 2 to 3.

The bi-monthly satellite data sets were merged to a monthly temporal resolution by averaging the two composites in the same month and bi-linearly remapped to the resolution of the applied reanalysis product (0.5° × 0.5°, CRU TS4.01).

1.1 Environmental driver variables

2.2 Environmental driver variables

We use temperature, precipitation and data time series of temperature and CO$_2$ to derive the observed historical forcing (Sect. 2.4) and climatologies of precipitation and temperature to calculate climatic regimes (Fig. 2). Monthly averages of near-surface air temperature and precipitation are from the latest version of the Climatic Research Unit Timeseries dataset (CRU TS4.01). The global data are gridded to 0.5° × 0.5° resolution (Harris et al., 2014). Global monthly means of atmospheric CO$_2$ concentration are from the GLOBALVIEW-CO2 product (obspack_co2_1_GLOBALVIEWplus_v2.1_2016_09_02; for details see https://doi.org/10.25925/20190520) provided by the National Oceanic and Atmospheric Administration / Earth System Research Laboratory (NOAA / ESRL).

2.3 Earth system model simulations

We analyzed recent climate-carbon simulations of seven ESMs participating in the fifth phase of the Coupled Model Inter-comparison Project, CMIP (Taylor et al., 2012). The model simulated data were obtained from the Earth System Grid Federation, ESGF (https://esgf-data.dkrz.de/projects/esgf-dkrz/). Seven ESMs provide output for the variables of interest (GPP, CO$_2$, LAI, and near-surface air temperature) for simulations titled esmHistorical, RCP4.5, RCP8.5, 1pctCO2, esmFixClim1, and esmFdbk1. It is the same set of models analyzed in Wenzel et al. (2016) and Wenzel et al. (2016) and Winkler et al. (2019). The individual model setups and components are illustrated in more detail in various studies, such as Arora et al. (2013); Wenzel et al. (2014); Mahowald et al. (2016); Winkler et al. (2019).

The esmHistorical simulation spanned the period 1850 to 2005 and was driven by observed conditions such as solar forcing, emissions or concentrations of short-lived species and natural and anthropogenic aerosols or their precursors, land use, anthropogenic as well as volcanic influences on atmospheric composition. The models are forced by prescribed anthropogenic CO$_2$ emissions, rather than atmospheric concentrations. CO$_2$ concentrations.
Several Representative Concentration Pathways (RCPs) have been formulated describing different trajectories of greenhouse gas emissions, air pollutant production and land use changes for the 21st century. These scenarios have been designed based on projections of human population growth, technological advancement and societal responses (van Vuuren et al., 2011; Taylor et al., 2012). We analyzed simulations forced with specified concentrations of a high emissions scenario (RCP8.5) and a medium mitigation scenario (RCP4.5) reaching a radiative forcing level of 8.5 and 4.5 W m$^{-2}$ at the end of the century, respectively. These simulations were initialized with the final state of the historical runs and spanned the period 2006 to 2100.

$\text{1pctCO}_2$ is an idealized fully coupled carbon-climate simulation initialized from a steady state of the preindustrial control run and atmospheric CO$_2$ concentration prescribed to increase 1% yr$^{-1}$ until quadrupling of the preindustrial level. The simulations esmFixClim and esmFdbk are set up similar to the aim to disentangle the two carbon cycle feedbacks in response to rising CO$_2$ analogous to the $\text{1pctCO}_2$ with the difference, that in esmFixClim (esmFdbk) only the radiative effect from increasing concentration is included. In esmFixClim CO$_2$-induced climate change is suppressed (i.e., radiation transfer model sees constant pre-industrial CO$_2$ level), while the carbon cycle sees the preindustrial level (vice versa) (Taylor et al., 2009, 2012) responding to increasing CO$_2$ concentration (vice versa for esmFdbk; Taylor et al., 2009, 2012; Arora et al., 2013).

### 2.4 Estimation of greening sensitivities

We largely follow the methodology detailed in Winkler et al., (2019). For both model and observational data, the two-dimensional global fields of LAI and the driver variables are cropped according to different classification schemes (namely, vegetation classes (Olson et al., 2001); climatic regimes, and latitudinal bands) (namely, climatic regimes, latitudinal bands and vegetation classes; Olson et al., 2001). The aggregated values are area-weighted, averaged in space, and temporally reduced to annual estimates dependent on the variable: annual maximum LAI, annual average atmospheric CO$_2$ concentration, and growing degree days (GDD0, yearly accumulated temperature of days where near-surface air temperature $> 0^\circ$C).

We use a standard linear regression model to derive the greening sensitivity. On a historical greening sensitivities in models and observations alike (for details see the Methods section Estimation of historical LAI$_{\text{max}}$ sensitivity in Winkler et al., 2019). On the global scale, LAI$_{\text{max}}$ is assumed to be a linear function of atmospheric CO$_2$ concentration. For the temperature-limited high northern latitudes, we also have to account for warming and include temperature as an additional driver. We do this using GDD0. We derive the dominant mode (denoted $\omega$) through a principal component analysis of (PCA) of CO$_2$ and GDD0 to avoid redundancy from co-linearity between the two driver variables, but retain their underlying time-trend and interannual variability. Thus, NHL (for details see the Methods section Dimension reduction using principal component analysis in Winkler et al., 2019). In particular, the PCA is performed on large-scale aggregated values as well as on pixel level to investigate on spatial variations. We only retain the first principal component (denoted $\omega$), which explains a large fraction of the variance in models and observations (for more details see Supplementary Table 1 in Winkler et al., 2019). Figure A1 depicts the temporal development of CO$_2$ and GDD0 as well as their principal component $\omega$ for observations. For the NHL, LAI$_{\text{max}}$ is then formul-
lated as a linear function of the proxy driver time series \( \omega \) (Winkler et al., 2019). The best-fit gradients and associated standard errors of the linear regression model represent the \( \text{LAI}_{\text{max}} \) sensitivities, or greening sensitivities, and their uncertainty estimates, respectively. For variations of finer spatial scale, the greening sensitivity is similarly calculated at the pixel scale.
3 Results and Discussion

There are two parts to the EC methodology (Fig. 4) – a statistically robust relationship between modeled matching pairs of predictor-predictand values and an observed value of the predictor. The predictors are from a representative historical period. The predictands are modeled changes in a variable of interest at a potential future another forcing state of the system – typically one that is difficult to measure (e.g. potential future). The projection of the observed predictor on the modeled relation yields a constrained value of the predictand. A causal basis has to buttress the predictor-predictand relationship, else the EC method may be spurious. For example, meaningful coupling between concurrent changes in GPP and LAI\text{max} with increasing atmospheric CO\textsubscript{2} concentration underpins our specific case study in the NHL, i.e. some of the enhanced GPP due to rising CO\textsubscript{2} concentration is invested in additional green leaves by the plants (Winkler et al., 2018). This plants (Myneni et al., 1997a; Forkel et al., 2016; Zhu et al., 2016; Mao et al., 2016; Winkler et al., 2019). Supplementary Figure 1 in Winkler et al. (2019) illustrates the specifics of the causal link underlying this predictor-predictand relationship. This tight coupling assures an approximately constant ratio of predictand to predictor across the models within the ensemble, thus setting up the potential for deriving an EC estimate.

Uncertainty on the constrained estimate depends on the observed predictor and modeled relationship, aside from the goodness-of-fit of the latter (Fig. 4). These are detailed below.

3.1 Uncertainty in Observed Sensitivity Predictor Due to Data Source

We investigate this observational uncertainty using LAI data from two different sources, AVHRR (1/12 degree) and MODIS (1/20 degree), and spatially aggregating these by broad vegetation classes, latitudinal bands and climatic regimes. The observed large-area sensitivities, large-scale LAI\text{max} sensitivities to CO\textsubscript{2} forcing are always positive (greening), irrespective of the source data and the method of aggregation (Fig. 2, Tab. 1). This indicates a net increase in green leaf area across the NHL during the observational period, as reported previously (Myneni et al., 1997a; Zhu et al., 2016; Forkel et al., 2016). Overall, MODIS based estimates have higher uncertainty because of the shorter length of the data record (17 years). The failure to reliably estimate sensitivities, sensitivities in tropical forests (also in the latitudinal band 30\degree S – 30\degree N, and in hot, wet and humid climatic regimes, see Tab. 1 and Fig. 2) is due to saturation of optical remote sensing data over dense vegetation (LAI\text{max} > 5) and problems associated with high aerosol content and ubiquitous cloudiness. In general other regions, the estimated sensitivities are comparable across sensors and aggregation schemes (e.g. for latitudinal band, in particular in the high latitudinal band (> 60\degree N/S, AVHRR: [3.4 ± 0.5] \times 10^{-3}, MODIS: [3.6 ± 0.9] \times 10^{-3} ; LAI\text{max} m^2 m^{-2} ppm^{-1}). However, there are three interesting exceptions. First, higher sensitivities are seen in croplands, which reflect management effects (fertilizer application, irrigation etc.) in addition to effects (Fig. 2a, Tab. 1). Second, lower sensitivities are seen in sparsely vegetated areas and biomes (low LAI\text{max}, < 1) which are due to nutritionally poor soils and / or inhospitable climatic conditions. Third, similarly low sensitivities are seen in dry regimes where precipitation is limiting and in humid regimes where temperature is limiting (Fig. 2c, Tab. 1: CO\textsubscript{2}). This aligns with previous studies reporting a net increase in green leaf area
across the high latitudes during the observational period (Myneni et al., 1997b; Zhu et al., 2016; Forkel et al., 2016).

This analysis illustrates the applicability and limitations of using observed greening sensitivities to CO₂ forcing as a constraint on photosynthetic production. For example, data from both AVHRR and MODIS sensors provide a comparable estimate of greening sensitivity in the colder high latitudes (boreal forests and tundra vegetation classes) where precipitation is generally less than 1000 mm (Winkler et al., 2018). However, the remote-sensing LAI data are not suitable for similar studies in areas dominated by croplands and in the tropics for reasons stated above (boreal forests and tundra vegetation classes; Winkler et al., 2019).

In the lower latitudes, however, the discrepancies among the two sensors indicate a considerable observational uncertainty and thus no robust estimation of the observed predictor is possible.

3.2 Uncertainty in Sensitivities Due to Spatial Aggregation

We focus further analyses on the NHL region (> 60° N; Fig. 2b) only because data from both AVHRR and MODIS sensors yield comparable spatially-aggregated greening sensitivities in this region unlike elsewhere, as discussed in Sect. 3.1, because of two reasons. First, the direct human impact (i.e., land management) can be neglected in the high latitudes, thus, we can assume that the observed changes reflect the response of natural ecosystems. Second, the observational evidence of an increased plant productivity in the recent decades is well established (e.g., Keeling et al., 1996; Myneni et al., 1997a; Graven et al., 2013; Forkel et al., 2016; – an important requisite in defining a robust predictor.

In addition to the physiological effect of CO₂, also warming plays a key role in controlling plant productivity of these the NHL temperature-limited ecosystems, and thus, vegetation greenness. To avoid redundancy from co-linearity between CO₂ and GDD0, we reduce dimensionality by performing a principal component analysis of the two driver variables (Sect. 2.4). The resulting first principal component explains most of the variance and retains the trend and year-to-year fluctuations in both CO₂ and GDD0. Therefore, we obtain a proxy driver (hereafter denoted ω) that represents the overall forcing signal causing observed vegetation greenness changes in NHL (Fig. A1). Accordingly, greening sensitivity for the entire NHL area is derived as response to ω, the combined forcing signal of rising CO₂ and warming. This procedure also enables a better comparability between observations and models because varying strengths of physiological and radiative effects of CO₂ among models are taken into account (Sect. 3.3 – 3.5).

The vegetated landscape in the NHL region is heterogeneous, with boreal forests in the south, vast tundra grasslands to the north and shrublands in-between. The species within each of these broad vegetation classes respond differently to changes in key environmental factors. Even within a species, such responses might vary due to different boundary conditions, such as topography, soil fertility, micrometeorological conditions, etc. How this fine scale variation in greening sensitivity impacts the aggregated value is assessed below.
The distribution of greening sensitivities from all NHL pixels is slightly skewed towards the positive (blue histogram). The mean value of this distribution (blue dashed line) is comparable to the sensitivity estimate derived from the spatially-averaged NHL time series (yellow dashed line; Fig. 3). Based on the Mann-Kendall test \((p > 0.1)\), nearly over half the pixels \((54\%)\) show positive statistically significant trends (greening), while about 10\% show browning trends \((\text{possibly due to disturbances; Goetz et al., 2005})\). The distribution of these statistically significant sensitivities (red histogram) therefore has two modes, a weak browning and a dominant greening mode, resulting in a substantially higher mean value (red dashed line) in comparison to the spatially-averaged estimate (yellow dashed line; Fig. 3). Thus, by taking into account the remaining 36\% of non-significantly changing pixels (as in the NHL spatially-averaged estimate), an additional source of uncertainty is possibly introduced. The mean sensitivity value is, of course, higher when only pixels showing a greening trend are considered in the analysis (green dashed line; Fig. 3). These are the only areas in NHL that actually show a large increase in plant productivity and consequently significant changes in leaf area.

Model output of several ESMs (CMIP5) reveal similar pixel-level variation in both the predictor \((\text{LAI}_{\text{max}}\text{ sensitivity to } \omega, \text{ historical simulation}; \text{ Sect. 2.3})\) and associated changes in GPP in the NHL \((\text{Anav et al., 2013, 2015})\), the predictand \((\text{GPP, IpctCO2}; \text{ Sect. 2.3})\), although ESMs operate on much coarser resolution \((\text{Fig. A2; see also Anav et al., 2013, 2015})\). Due to the coupling of the predictor and predictand, the distribution of all pixel estimates-pixels with significant changes is approximately the same for the two variables \((\text{Fig. A2})\). Accordingly, averaging the equally distributed estimates likely does not affect the predictor-predictand relationship in the model ensemble (Fig. 1). Consequently, if all spatial gridded data arrays are consistently processed to spatially-aggregated estimates, each predictand and predictor (observed and modeled) estimate contain a coherent component of spatial variations. In other words, considering browning and non-significant pixels results in a lower overall \(\text{LAI}_{\text{max}}\) sensitivity in NHL, which in turn leads to a lower constrained estimate of \(\Delta\text{GPP}\) in NHL. This is consistent with the underlying relationship between predictor and predictand. On a related note, Bracegirdle and Stephenson \((2012a)\), Bracegirdle and Stephenson \((2012a)\) suggest that this source of error is not significantly dependent on the spatial resolution when comparing model subsets from high to low resolution.

The above analysis informs that spatially-averaged estimates are approximations containing a random error component due to inclusion of data from insignificantly changing pixels and a systematic bias component from browning pixels of reversed sign. This uncertainty is relevant to the EC method, where the observed sensitivity decisively determines the constrained estimate from the ensemble of ESM projections \((\text{Kwiatkowski et al., 2017})(\text{Kwiatkowski et al., 2017; Winkler et al., 2019})\). However, if spatial variations are treated consistently as an inherent component of observations and models, the EC method is only slightly susceptible to this source of uncertainty.
3.3 Uncertainty in Sensitivities Due to Temporal Variations

We seek recourse to longterm CMIP5 ESM simulations covering the historical period 1850 to 2005 (Sect. 2.3) to assess temporal variation in the predictor variable, because of the shortness of observational record. Three representative models (CESM1-BGC, MIROC-ESM, and HadGEM2-ES) spanning the full range of NHL greening sensitivities in the CMIP5 ensemble (Winkler et al., 2018) (Winkler et al., 2019) are selected for this analysis. For each model, LAI$_{\text{max}}$ sensitivity to $\omega$ in moving windows of different lengths are evaluated (15, 30, and 45 years; Fig. 4) are evaluated and A3). The analysis reveals two crucial aspects that highlight how temporal variations impair comparability of the predictor variable between models and observations – an essential component of the EC approach.

First, window locations of modeled and observed predictor variable have to match. If the forcing in the simulations is low, for example, as in the second half of the 19th century when CO$_2$ concentration was increasing slowly, inter-annual variability dominates and LAI$_{\text{max}}$ sensitivity cannot be accurately estimated irrespective of the window length (Fig. 4 and A3). With increasing forcing over time (rising yearly rate of CO$_2$ infusion, and consequently, the concentration), the signal-to-noise ratio increases and LAI$_{\text{max}}$ sensitivity to $\omega$ estimation stabilizes, for example, as in the second half of the 20th century. Therefore, LAI$_{\text{max}}$ sensitivities estimated at different temporal locations result in non-comparable values and eventually a false constrained estimate (details in Sect. 3.4). As an example, modeled sensitivities based on a 30-year window centered on year 1900, when CO$_2$ level increased by 10 ppm, with and observed sensitivity estimated from a 30-year window centered on year 2000, when CO$_2$ level increased by 55 ppm, describe different states of the system and therefore should not be used contrasted in the EC method.

Second, in addition to temporal location, also window lengths have to match between observations and models. For all three models, sensitivities estimated from 15-year chunks show high variability and thus, a 15-year record is perhaps too short to obtain robust estimates. The LAI$_{\text{max}}$ sensitivity estimation becomes more stable with strengthening forcing and increasing window length (Fig. 4 and A3). As a consequence, using short-term observed sensitivity as a constraint on long-term model projections results in an incorrect EC estimate. Hence, the MODIS sensor record is, on the one hand, too short and does not, on the other hand, overlap temporally with the historical CMIP5 forcing. Therefore, it does not provide a correct observational constraint robust predictor in this EC study.

3.4 Level and Time Rate of CO$_2$ Forcing

The EC method raises an obvious question – does it not implicitly assume that the key operative mechanisms underpinning the EC relation remain unchanged because a future system state is being predicted based on its past behavior? To be specific, we are attempting to predict GPP at a future point in time based on greening sensitivity inferred from the past. Does this not require the assumption that the key underlying relationship which makes this prediction possible, namely, a robust coupling between contemporaneous changes in GPP and LAI$_{\text{max}}$ remains unchanged from the past to the future? To address this question, we
resort to the CMIP5 idealized simulation (1pctCO2), where atmospheric CO$_2$ concentration increases 1% annually, starting from a pre-industrial level of 284 ppm until a quadruple of this value is reached (Sect. 2.3). We limit the analysis to the three models (CESM1-BGC, MIROC-ESM, and HadGEM2-ES) which bracket the full range of GPP enhancement and LAI$_{\text{max}}$ sensitivity in the original seven ESM ensemble (Winkler et al., 2018) (Winkler et al., 2019).

The relationship between simultaneous changes in GPP and LAI$_{\text{max}}$ remains linear for all CMIP5 models in the range 1\times CO$_2$ to 2\times CO$_2$ (Fig. 5 and A4, Tab. 2). With concentration increasing beyond 2\times CO$_2$, all models show weakening correlation ($R^2$, Tab. 2) and decreasing slope ($b$, Tab. 2) of this relationship (Fig. 5 and A4), suggesting a saturating rate of allocation of additional GPP to new leaves at higher levels of CO$_2$. Consequently, LAI$_{\text{max}}$ sensitivity to increasing CO$_2$ and associated warming decreases. At and over 4\times CO$_2$ (1140 ppm), a level unlikely to be seen in the near future, there appears to be no relationship between ΔGPP and ΔLAI$_{\text{max}}$. This raises the question as to what extent does the weakening of the relationship between the predictor and predictand (Fig. 1) at higher concentration affect the EC analysis (Fig. 1). To shed light on this matter, we perform the following Gedankenexperiment thought experiment.

Understanding the relationship and interplay between forcing (increasing CO$_2$ concentration), predictor (LAI$_{\text{max}}$ sensitivity), and the predictand (ΔGPP) is key to evaluating the EC method. We conceive four possible scenarios of how the system might behave with increasing forcing. For simplicity, we assume linearly increasing concentration, use LAI instead of CO$_2$ concentration, LAI represents LAI$_{\text{max}}$, and GPP refers to its annual value below (Fig. 6). The four scenarios are: All linear, all non-linear (saturation), and two mixed linear / non-linear cases (Tab. A1). We emulate a multi-model ensemble by applying different random parameterizations for the linear and saturation (the hyperbolic tangent function) responses of GPP to CO$_2$ and of LAI to GPP. One of these realizations is assumed to represent pseudo-observations (dashed lines, Fig. 5). We discuss one case in detail for illustrative purposes (No. 3, Tab. A1).

In scenario 3, ΔGPP increases linearly with increasing CO$_2$ (Fig. 6a), while ΔLAI/ΔGPP saturates (Fig. 6b). The LAI sensitivity to CO$_2$ weakens with increasing forcing (Fig. 6c) as a response to saturation of GPP allocation to leaf area. We derive LAI sensitivities to CO$_2$ for three different periods (‘past periods’ in Fig. 6c) to constrain ΔGPP at a much higher CO$_2$ level (‘projected period’ in Fig. 6a). Next, we apply the EC method on these pseudo-projections of ΔGPP relying on LAI sensitivities derived from the three past periods (Fig. 6d). The EC method is applicable even at a low forcing level (past period 1) in this simplified scenario because we neglect stochastic internal variability of the system. The slope of emergent linear relationship increases (Fig. 6d) as modeled LAI sensitivities decrease with rising CO$_2$ concentration (Fig. 6c). The observational constraint on future ΔGPP, however, remains nearly the same, because pseudo-observed LAI sensitivity also weakens at higher CO$_2$ levels (dashed lines, Fig. 6c, d). Thus, the three EC estimates of ΔGPP are approximately identical (Fig. 6d) and independent of the forcing level during past periods. With intensified forcing, the relationship between predictor and predictand remains linear within the model ensemble, although their relationship becomes non-linear within each model and, crucially, in reality as well. In other words, as long as the models agree on the occurrence and timing of saturation, changes in strength of saturation for
Given forcing, i.e., the dynamics of the system, the inter-model variations of predictor and predictand relate linearly within the model ensemble—ensemble (Fig. 6). The same behavior is also seen in the other three scenarios (Tab. A1; Fig. A5, A6).

Nevertheless, with ever increasing forcing and associated steepening of the emergent linear relationship, the LAI sensitivity loses its explanatory power at some point because the linear relationship eventually lies within the observational uncertainty and no meaningful constraint can be derived. This and disagreement between models on system dynamics are ultimate limits of the EC method. Interestingly, we find that all CMIP5 models agree on the occurrence of saturation, but slightly disagree on the timing of saturation—strength of saturation for given CO₂ forcing (Fig. 5, A4, and Tab. 2). Further, we find that the 'all non-linear' scenario best describes the dynamics of the system in the forcing range from 1×CO₂ to 4×CO₂. However, the saturation of LAI to GPP happens at a lower CO₂ level than saturation of GPP to CO₂. Still, inferences from interpretation of Case 3 (Fig. 6) are equally applicable.

Results from the above Gedankenexperiment thought experiment also highlight the importance of matching window locations and lengths between models and observations, as discussed earlier (Sect. 3.3). For instance, taking LAI sensitivity from past period 2 (green dashed line, Fig. 6d) as an observational constraint on the multi-model linear relationship based on past period 3 (red solid line, Fig. 6d), results in a significant overestimation of constrained ΔGPP (intersection of the two lines, Fig. 6d).

The above analysis informs that the constrained GPP estimate at one future period (e.g., 2×CO₂) is nearly independent of the past periods from when the observational sensitivities are derived, for most realistic scenarios. Now, we evaluate the EC method where sensitivity from one past period is used to obtain constrained GPP estimates at different periods in the potential future, i.e. progressively farther down the time-line of a CO₂-enriched world. We utilize the greening sensitivity derived from 35 years of observed LAI_max data (AVHRR, Sect. 2.1.1) and apply the EC method to CMIP5 1pctCO2 simulations. The sensitivities in this case are due to forcing from both CO₂ increase and associated warming during the observational period (Sect. 2.4). We seek constrained GPP estimates at future for the NHL at different CO₂ levels (2×CO₂, 3×CO₂, and 4×CO₂).

Winkler et al. (2019) previously reported a strong linear relationship between modeled contemporaneous changes in LAI_max and GPP arising from the combined radiative and physiological effects of CO₂ enrichment until 2×CO₂ in the CMIP5 ensemble (Fig. 5). As a result, models with low LAI_max sensitivity to CO₂ project lower ΔGPP for a given increment of CO₂ concentration, and vice versa. Thus, the large variation in modeled historical LAI_max sensitivities linearly maps to variation in ΔGPP at 2× (Winkler et al., 2018; blue line, Fig. 7a). CO₂ (Winkler et al., 2019, blue line, Fig. 7a). At higher levels, such as 3×CO₂ (green line, R² = 0.93) and 4×CO₂ (red line, R² = 0.88), this linear relationship within the model ensemble, while still present, weakens (Fig. 7a; Tab. 3). This is because the CMIP5 models do not agree on the timing and magnitude strength of the saturation effect at higher CO₂ levels (Fig. 7a5 and A4). The increment in constrained GPP estimates for successive equal increments of CO₂ decreases due to the saturation effect in all CMIP5 models (dashed horizontal lines, Fig. 7a). For example,
the change in GPP between $3 \times \text{CO}_2$ and $4 \times \text{CO}_2$ ($\Delta \text{GPP} \sim 1.06 \text{ Pg C yr}^{-1}$, Tab. 3) is much lower than between $2 \times \text{CO}_2$ and $3 \times \text{CO}_2$ ($\Delta \text{GPP} \sim 2.34 \text{ Pg C yr}^{-1}$, Tab. 3).

We have thus far focused on the magnitude of $\text{CO}_2$ concentration change and not on the time rate of this change. For example, a given amount of change in $\text{CO}_2$ concentration, say 200 ppm, can be realized over different time periods, say over a 100 or 150 years. The problem of varying rates of $\text{CO}_2$ concentration change is implicitly encountered when ESMs are executed under different forcing scenarios, such as RCPs (Sect. 2.3). A question then arises whether the constrained GPP predictand estimate is independent of the time rate of $\text{CO}_2$ concentration change and dependent only on the magnitude of $\text{CO}_2$ concentration change. To investigate this aspect of forcing, we extract GPP estimates at the same $\text{CO}_2$ concentration (535 ppm; final concentration in RCP4.5) from three simulations of different forcing rates and calculate the difference relative to a common initial $\text{CO}_2$ concentration (380 ppm; initial concentration of RCP scenarios). Hence, the magnitude of the forcing is the same but applied over different durations (RCP4.5: ~90yr, RCP8.5: ~45yr, and 1pctCO2: ~30yr). A clear majority of the CMIP5 models show substantial differences in $\Delta \text{GPP}$ between the different pathways of $\text{CO}_2$ forcing. In general, GPP changes are higher for lower time rates of $\text{CO}_2$ forcing, i.e. forcing over longer time periods. As a consequence, the EC estimates of $\Delta \text{GPP}$ for the same increase in $\text{CO}_2$ concentration are scenario-dependent (Fig. 7b; Tab. 3) – a counter-intuitive result. For instance, $\Delta \text{GPP}$ in the low-$\text{CO}_2$-rate scenario (RCP4.5: $\Delta \text{GPP} \sim 2.84 \text{ Pg C yr}^{-1}$, Tab. 3) is $\sim 39\%$ (1pctCO2: $\Delta \text{GPP} \sim 2.05 \text{ Pg C yr}^{-1}$, Tab. 3) and $\sim 20\%$ (RCP8.5: $\Delta \text{GPP} \sim 2.38 \text{ Pg C yr}^{-1}$, Tab. 3) higher than the high-$\text{CO}_2$-rate scenarios for an increase of 155 ppm $\text{CO}_2$. This analysis suggests that the vegetation response to rising $\text{CO}_2$ is pathway dependent, at least in the NHL. One of the reasons for this could be species compositional changes in scenarios of low forcing rates, i.e. over longer time frames. This novel result, however, requires a separate in-depth study.

### 3.5 Effects of $\text{CO}_2$ Forcing

Higher concentration of $\text{CO}_2$ in the atmosphere stimulates plant productivity through the fertilization and radiative effects (Ne- mani et al., 2003; Leakey et al., 2009; Arora et al., 2011; Goll et al., 2017). The two effects can be disentangled in the model world by conducting simulations in a 'fertilization effect only' (esmFixClim1) and a 'radiative effect only' (esmFdbk1) setup (Sect. 2.3). These are termed below as idealized model simulations. We investigate here whether historical runs and observations, which include both effects, can be used to constrain GPP changes in idealized CMIP5 simulations (e.g. as in Wenzel et al., 2016).

We find strong linear relationships between historical LAI$_\text{max}$ sensitivity and $\Delta \text{GPP}$ for $2 \times \text{CO}_2$ in both idealized setups (esmFixClim1: $R^2 = 0.92$, esmFdbk1: $R^2 = 0.98$, Tab. 3, Fig. 7c). Consequently, this linear relationship is also pronounced for calculated sums of both effects for each model (esmFixClim1 + esmFdbk1: $R^2 = 0.95$, Tab. 3, Fig. 7c). This suggests that the two effects act additively on plant productivity and, thus, each effect can be simply expressed in terms of a scaling factor of the total GPP enhancement. Hence, the application of the EC method on idealized simulations using real world observations is
Interestingly, the two effects contribute about the same to the general increase in GPP at $2 \times \text{CO}_2$ (esmFixClim1: $\Delta \text{GPP}$ $\sim$ 1.35 Pg C yr$^{-1}$, esmFdbk1: $\Delta \text{GPP}$ $\sim$ 1.38 Pg C yr$^{-1}$, Tab. 3, Fig. 7c). At higher concentrations, such as $3 \times \text{CO}_2$ and $4 \times \text{CO}_2$, the enhancement in GPP saturates in both idealized setups. However, the radiative effect becomes dominant relative to the CO$_2$ fertilization effect when CO$_2$ concentration exceeds $2 \times \text{CO}_2$ (e.g. at $4 \times \text{CO}_2$, esmFixClim1: $\Delta \text{GPP}$ $\sim$ 2.42 Pg C yr$^{-1}$, esmFdbk1: $\Delta \text{GPP}$ $\sim$ 3.06 Pg C yr$^{-1}$, Tab. 3). Therefore, we can expect that at some point in the future, NHL photosynthetic carbon fixation will benefit more from climate change (e.g. warming) than from the fertilizing effect of CO$_2$.

### 3.6 Uncertainties in the multi-model ensemble

Besides methodological sources of uncertainty discussed above, the estimate of an EC may also be deficient due to inaccurate assumptions about the model ensemble. First, possible common systematic errors in a multi-model ensemble (i.e. the entire ensemble misses an unknown but for the future essential process) are implicitly omitted in the EC approach, however, could cause a general over- or underestimation of the constrained value (Bracegirdle and Stephenson, 2012b; Stephenson et al., 2012). Second, the set of forcing variables for historical simulations may be incomplete (i.e. not yet identified drivers of observed changes) and, thus, the comparability of observations and model simulations is limited (Flato et al., 2013). Third, the EC method can be overly sensitive to individual models of the ensemble, which has a bearing on the robustness of the constrained value (Bracegirdle and Stephenson, 2012b). Bracegirdle and Stephenson (2012b) proposed a diagnostic metric (Cook’s distance) to test an ensemble for influential models. Fourth, the assumption behind the predictand-predictor relationship not only has to rely on a physical, but also on a logical connection within the model ensemble, meaning that the analyzed characteristic of the predictor variable (e.g. sensitivity to the forcing, or historical relative/absolute changes) is causally linked to changes in the predictand variable. For instance, Wenzel et al. (2016) reported Wenzel et al. (2016) established a linear relationship between relative changes in the predictand taking the initial state into account (changes in GPP for doubling of CO$_2$ relative to the preindustrial state, and initial pre-industrial state), and a predictor neglecting the initial state (historical sensitivity of CO$_2$ amplitude to rising $\Delta$CO$_2$). This statistical relationship can be spurious, because the model skill of simulating an accurate initial state and a plausible sensitivity to a forcing are not connected.

These issues are to be contemplated when establishing an EC estimate and evaluating its robustness.

### 4 Conclusions

An in-depth analysis of the EC method is illustrated in this article through its application to projections of change in NHL photosynthesis under conditions of rising atmospheric CO$_2$ concentration. Key conclusions highlighting the functionality of the EC method are presented below.
The importance of how the observational predictor is obtained cannot be emphasized enough because it essentially defines the constrained estimate. Thus, considerable care is required when selecting and processing the observational datasets. The LAI data products of both AVHRR and MODIS sensors provide comparable estimates of greening sensitivity in the colder northern high latitudes (i.e. boreal forests and tundra vegetation classes). In these ecosystems, factors associated with GPP enhancement from forcing and consequent investment in leaf area dominate. This is not the case in croplands and tropical areas. Therefore, the use of greening sensitivity as an observational constraint is not feasible in regions where croplands and/or tropical vegetation dominate. The EC method is particularly sensitive to observational uncertainty. The single observational estimate essentially determines the EC, whereas the emergent linear relationship is established based on a collection of multi-model estimates (each model gets 'one vote', however, some models might be more influential than others; Bracegirdle and Stephenson, 2012b). Hence, the observational uncertainty has a much larger bearing on the EC than the uncertainty of each individual model. To overcome this source of uncertainty, various meaningful observations should be taken into consideration when establishing the observed predictor.

Spatially aggregating observations and model output of different resolutions in the EC method constitutes another source of uncertainty. Regional estimates of greening sensitivity Predictors and predictands expressed as regional estimates (e.g. area-weighted mean of the NHL) are approximations of complex fine-scale processes. Aggregation will inevitably introduce a random error component due to inclusion of data estimates from areas where LAI the predictor is not changing and or a systematic bias from areas where LAI is decreasing (browning). The predictor has a reversed sign. Thus, the spatially-aggregated greening sensitivity is variables are meaningful only if most of the region is greening in response to forcing. However, in agreement about the response to CO2 forcing (e.g. more than half of the NHL is greening with rising CO2). However, we find that the source of uncertainty related to spatial aggregation is of minor importance as long as spatial variations in observations and models simulations are treated consistently, this source of uncertainty is likely of minor importance.

A large source of uncertainty is associated with temporal variability of the predictor variable throughout the historical period. The evaluation of greening sensitivity requires when comparing models and observations. Establishing a robust predictor requires evaluating temporal window lengths of sufficient duration (approximately 30 years, and location) and their locations along the forcing time line. And, these window length and location should match between models and observations in the EC method. For example, the analysis in Wenzel et al. (2016) Wenzel et al. (2016) might have yielded different results and conclusions if model and observational predictor sensitivities were temporally matched. The We find that the relevance of window length decreases with increasing and accelerating forcing, depending on the magnitude of natural/external variability (signal-to-noise ratio) of the predictor variable.

The level, effect and duration of forcing time-rate of applied CO2 forcing can have a bearing on the linear relationship between GPP enhancement and predictor sensitivities the predictand and predictor variables (Fig. 1). For example1, In our case study, the relationship underpinning the EC method, namely, that between concurrent ΔGPP and ΔLAI max changes changes
non-linearly with increasing forcing level (i.e. saturation with rising CO₂ concentration). This relation breaks down. The EC method can still be applied, because the CMIP5 models agree on the non-linear behavior of the system. However, at very high concentrations CO₂ concentrations the models diverge and this relation breaks down, at which point the EC method fails. The two dominant effects of rising CO₂ concentration on vegetation, namely, the fertilization and radiative effects, appear to be approximately additive in terms of GPP enhancement to forcing CO₂ forcing in the NHL. Therefore, the EC method can be applied to constrain estimates of GPP due to one or the other, or both the effects. The models, however, document a higher radiative effect than fertilization at high CO₂ concentrations, i.e. 3 × CO₂ and higher. An intriguing conclusion from our analysis is that the time-rate of forcing has an effect on GPP changes, that is, the projected GPP enhancement to CO₂ forcing seems to be dependent on how the forcing is applied over time, as in different scenarios or RCPs. This aspect is presently not well understood and requires further study.

The analyses and inferences presented in this article lead to the following concrete result. The uncertainty on EC estimate of GPP enhancement in NHL (ΔGPP = +3.4 Pg C yr⁻¹) for a doubling of pre-industrial atmospheric concentration is ± 0.2 Pg C yr⁻¹ (Winkler et al., 2018). This EC estimate is 60% larger than the conventionally used average of model projections (44% higher at the global scale), leading to conclude that most CMIP5 models included in their analysis were largely underestimating photosynthetic production. EC framework is widely promoted as observation-based evaluation tool for climate projections, especially in the context of the nascent CMIP6 ensemble (Eyring et al., 2019; Hall et al., 2019). Previous EC studies, however, exclusively focused on predictor-predictand combinations which exhibit so-called existent ECs (Hall et al., 2019), i.e. predictor and predictand are found to relate linearly across the ensemble. In the context of ESM evaluation, non-existent ECs, i.e. predictor and predictand are found to be unrelated in the ensemble, are equally important. Since predictor and predictand variables are premised on our mechanistic process understanding, non-existent ECs reveal a fundamental disagreement on the system dynamics among the models. This study encourages to scrutinize these system dynamics in the predictor-predictand space and also report such non-existent, yet expected, ECs in order to advance model development and evaluation.

Across different disciplines each EC and its set of predictor and predictand are unique to some extent and require an individual detailed examination. In this article, we scrutinized addressed general potential sources of uncertainty and limitations of the applicability of the EC method. Our findings are illustrated by in the EC method by the means of a case study in carbon cycle research, however, are generally relevant and applicable. Thus, the illustrated results are qualitatively transmissive to other sets of predictors and predictands and are generally relevant in Earth system sciences.
Author contributions. A.J.W. performed the analysis. All authors contributed ideas and to writing of the manuscript.

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References


Figure 1. Schematic depiction of the Emergent Constraint (EC) method and factors affecting the uncertainty of the constrained estimate. The predictor (x axis) is change in annual maximum of green leaf area index ($LAI_{\text{max}}$) due to unit forcing ($CO_2$ increase and associated climatic changes) during a representative historical period. It is termed greening sensitivity in this study. The predictand (y axis) is projected changes in Gross Primary Productivity (GPP) in response to rising $CO_2$ concentration (e.g. for a doubling of the pre-industrial level). Both the predictor and predictand refer to large area values, in this case, the entire Northern High Latitudes (NHL). Inter-model variations (each symbol represents a model) in matching pairs of predictor and predictand result in a linear relationship between the two (green band), i.e. the ratio (predictand/predictor) is approximately constant across the model ensemble. The slope depends on forcing attributes (gray shading), such as its level ($CO_2$ concentration, Sect. 3.4), time rate of application (scenarios such as various RCPs, Sect. 3.4) and different effects (i.e. fertilization, radiative, etc., Sect. 3.5). The observed sensitivity (yellow vertical bar) is used to find the constrained estimate of the predictand (i.e. change in GPP). The ability to accurately estimate the predictor depends on the source of observational data (Sect. 3.1), and its spatial (Sect. 3.2) and temporal variability (Sect. 3.3). Observed (yellow bar) and modeled predictor values (x coordinate of symbols) must be obtained from matching time periods, i.e. at the same level of historical forcing, to ensure comparability (Sect. 3.3 and 3.4). All these factors, together with the goodness-of-fit of inter-model variations (width of green shading), finally define the uncertainty of the derived constrained estimate (blue horizontal bar with black solid lines depicting the upper and lower bound of uncertainty).
Figure 2. Bar charts showing regression slopes of $LAI_{\text{max}}$ against atmospheric CO$_2$ concentration for broad vegetation classes (a; Olson et al., 2001; Fritz et al., 2015), Olson et al. (2001), latitudinal bands (b) and climate regimes (c). The class "Other" includes deserts, mangroves, barren and urban land, snow and ice, and permanent wetlands. The climatic boundaries are defined as follows - cold: < 10°C; warm: > 10°C & < 25°C; hot: > 25°C; dry: < 500 mm a$^{-1}$; wet: > 500 mm a$^{-1}$ & < 1000 mm a$^{-1}$; humid: > 1000 mm a$^{-1}$. Sensitivities evaluated from data from two satellite-borne sensors are shown, AVHRR (1982 – 2016; Pinzon and Tucker, 2014) and MODIS (2000 – 2016; Yan et al., 2016a, b). Grey bars indicate the standard error of the best linear fit.
Figure 3. Histograms and associated probability density functions (Gaussian kernel density estimation) of observed LAI$_{\text{max}}$ sensitivity to $\omega$ at pixel scale for the northern high latitudinal band ($> 60^\circ$ N, data from AVHRR sensor). Blue color depicts the distribution of LAI$_{\text{max}}$ sensitivities of all pixels and the red color for pixels with statistically significant (Mann-Kendall test, $p < 0.1$) greening or browning trends (the dashed lines denote the respective mean value). The green dashed line shows the mean value of 'greening' pixels only, whereas the yellow dashed line shows the LAI$_{\text{max}}$ sensitivity to $\omega$ for the entire northern high latitudinal belt.
Figure 4. Temporal variation of LAI\(_{\text{max}}\) sensitivity to \(\omega\) in three selected CMIP5 models spanning the full range from low (CESM1-BGC, a), to closest-to-observations (MIROC-ESM, b), to high-end (HadGEM2-ES, c). The colored lines show LAI\(_{\text{max}}\) sensitivity variations for moving windows of varying length of 15 (blue), 30 (green), and 45 (red) years over the historical period from 1860 to 2005.
Figure 5. Correlation of $\Delta L_{\text{AI}_{\text{max}}}$ and $\Delta \text{GPP}$ with increasing CO$_2$ forcing, starting from a pre-industrial concentration of 280 ppm (4x1xCO$_2$) to 4x4xCO$_2$ (CMIP5 1pctCO2 simulations). Results are shown for three selected CMIP5 models spanning the full range of $L_{\text{AI}_{\text{max}}}$ sensitivity to $\omega$, low-end: CESM1-BGC (a), closest-to-observations: MIROC-ESM (b), and high-end: HadGEM2-ES (c). Blue colored dots show the relation between 4x1xCO$_2$ and 2x2xCO$_2$, green colored dots between 2x2xCO$_2$ and 3x3xCO$_2$, and red colored dots between 3x3xCO$_2$ and 4x4xCO$_2$. The respective colored lines represent the best linear fit through those dots and the shading represents the 95% confidence interval.
Figure 6. Gedankenexperiment to examine the applicability of Emergent Constraints (EC) analysis under the assumption of an idealized linear / non-linear behavior of the system (Case 3, Table A1). a, Changes in GPP relate linearly to changes in CO₂ concentration. The yellow band marks the projection period of interest, i.e. the period of CO₂ concentration from \( x + 4\Delta \) to \( x + 5\Delta \). b, The increment in LAI with increasing GPP is assumed to decrease with rising CO₂ concentration (described by a hyperbolic tangent function). The parameterization in the linear and non-linear functions for pseudo observations (dashed black line) as well as models (solid grey lines) are determined randomly for each model. c, The diagnostic variable, LAI sensitivity to CO₂, is decreasing with increasing CO₂ as a consequence of the non-linear relation between ΔGPP and ΔLAI. The colored bands indicate three ‘past’ periods from \( x \) to \( x + \Delta \) (blue), \( x + \Delta \) to \( x + 2\Delta \) (green), and \( x + 2\Delta \) to \( x + 3\Delta \) (red). d, Linear relationships among the pseudo model ensembles (Ensemble LR, colored lines) between LAI sensitivities to CO₂ of the three past periods and ΔGPP from the projected period. Colored dots mark different models and the dashed lines represent associated pseudo observations for the respective historical period. Yellow solid line depicts the constant Emergent Constraint (EC) on projected ΔGPP irrespective of the past period.
Figure 7. Linear relationships between historical sensitivity of LAI$_{\text{max}}$ to $\omega$ and absolute increase of GPP at different levels (a), different time-rates (b) as well as effects of rising $\text{CO}_2$ (c). The black solid line depicts the observational sensitivity including the standard error (grey shading). Each CMIP5 model is represented by a distinct marker (legend at the top). The colored lines show the best linear fits including the 68% confidence interval estimated by bootstrapping across the model ensemble. The colored dashed lines indicate the derived constraints on $\Delta$GPP. a, Absolute changes in GPP at different levels of $\text{CO}_2$: 2$\times$CO$_2$ (blue), 3$\times$CO$_2$ (green), and 4$\times$CO$_2$ (red). b, Absolute changes in GPP for rising $\text{CO}_2$ concentration from 380 to 535 ppm at different time-rates: RCP4.5 (90 yr, blue), RCP8.5 (45 yr, green), and 1pctCO2 (30 yr, red). c, Absolute changes in GPP due to the two disentangled effects of $\text{CO}_2$ at 2$\times$CO$_2$ in idealized simulations: Fertilization effect (esmFixClim1, blue), radiative effect (esmFdbk1, green), and the sum of both effects (red).
Table 1. Coefficients of determination ($R^2$) of LAI$_{\text{max}}$ sensitivity to CO$_2$ for different large-scale aggregated regions. Data are from two optical remote sensors of different time length, AVHRR (1982 – 2016) and MODIS (2000 – 2016). Asterisks denote non-significant values: ** $p > 0.1$; * $p > 0.05$.

<table>
<thead>
<tr>
<th>Correlation coefficient</th>
<th>$R^2$</th>
<th>AVHRR</th>
<th>MODIS</th>
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<tr>
<td><strong>Biomes</strong></td>
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<tr>
<td>Boreal forests</td>
<td>0.49</td>
<td>0.58</td>
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<td>Temperate forests</td>
<td>0.47</td>
<td>0.81</td>
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<tr>
<td>Tropical forests</td>
<td>0.41</td>
<td>0.06**</td>
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</tr>
<tr>
<td>Graslands</td>
<td>0.75</td>
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<td>Croplands</td>
<td>0.75</td>
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<tr>
<td>Other</td>
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<tr>
<td><strong>Latitudinal Bands</strong></td>
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<td>&gt; 60° N/S</td>
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<td>0.21*</td>
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<td>warm dry</td>
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<td>0.36</td>
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<tr>
<td>warm wet</td>
<td>0.37</td>
<td>0.18*</td>
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</tr>
<tr>
<td>warm humid</td>
<td>0.25</td>
<td>0.12**</td>
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<tr>
<td>hot dry</td>
<td>0.08*</td>
<td>0.08**</td>
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<tr>
<td>hot wet</td>
<td>0.15</td>
<td>0.00**</td>
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<tr>
<td>hot humid</td>
<td>0.13</td>
<td>0.01**</td>
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Table 2. Slopes ($b$) and coefficients of determination ($R^2$) for regression between changes of $LAI_{max}$ against changes in annual mean GPP at different atmospheric CO$_2$ levels in all available CMIP5 models (1pctCO2 simulation). Asterisks denote non-significant values: ** $p > 0.1$; * $p > 0.05$.

<table>
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<tr>
<th>Correlation details</th>
<th>$&lt; 2xCO_2$</th>
<th>$&gt; 2xCO_2$ &amp; $&lt; 3xCO_2$</th>
<th>$&gt; 3xCO_2$</th>
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<tr>
<td></td>
<td>$b$</td>
<td>$R^2$</td>
<td>$b$</td>
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<tr>
<td>MIROC-ESM</td>
<td>0.23</td>
<td>0.97</td>
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<td>CESM1-BGC</td>
<td>0.45</td>
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<td>GFDL-ESM2M</td>
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<td>0.89</td>
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<td>CanESM2</td>
<td>0.22</td>
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<td>HadGEM2-ES</td>
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<td>0.99</td>
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<tr>
<td>MPI-ESM-LR</td>
<td>0.13</td>
<td>0.94</td>
<td>0.09</td>
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<td>NorESM1-ME</td>
<td>0.26</td>
<td>0.94</td>
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Table 3. Coefficients of determination ($R^2$) of the emergent linear relationships in Figure 7 (asterisks denote non-significant values: ** p > 0.1; * p > 0.05). Emergent Constraints ECs on ΔGPP (upper and lower bound of uncertainty in square brackets) for different atmospheric CO$_2$ levels and fully-coupled as well as idealized setups. The rightmost column shows the increase of ΔGPP for an increment of 1×CO$_2$. The lowermost section compares EC estimates of ΔGPP for equivalent changes in CO$_2$ concentration (CO$_2$ rises from 380 to 535 ppm), but for different time-rates.

<table>
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<tr>
<th></th>
<th>$R^2$</th>
<th>EC ΔGPP estimate (Pg C yr$^{-1}$)</th>
<th>EC ΔGPP for Δ1×CO$_2$ (Pg C yr$^{-1}$)</th>
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<td>2xCO$_2$</td>
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<tr>
<td>Fully coupled (1pctCO2)</td>
<td>0.96</td>
<td>3.36 [3.15, 3.56]</td>
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<tr>
<td>CO$_2$ fertilization only (esmFixClim1)</td>
<td>0.88</td>
<td>1.35 [1.29, 1.62]</td>
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<tr>
<td>Radiative effect only (esmFdbk1)</td>
<td>0.94</td>
<td>1.38 [1.13, 1.51]</td>
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</tr>
<tr>
<td>Sum of both effects (esmFixClim1 + esmFdbk1)</td>
<td>0.95</td>
<td>2.74 [2.6, 2.9]</td>
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<tr>
<td>3xCO$_2$</td>
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<tr>
<td>Fully coupled (1pctCO2)</td>
<td>0.93</td>
<td>5.7 [5.26, 6.16]</td>
<td>2.34</td>
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<tr>
<td>CO$_2$ fertilization only (esmFixClim1)</td>
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<td>2.15 [2.02, 2.37]</td>
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<td>Radiative effect only (esmFdbk1)</td>
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<td>2.53 [2.3, 2.66]</td>
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<td>Sum of both effects (esmFixClim1 + esmFdbk1)</td>
<td>0.96</td>
<td>4.68 [4.38, 4.97]</td>
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<td>4xCO$_2$</td>
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<td>Radiative effect only (esmFdbk1)</td>
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<td>3.06 [2.83, 3.2]</td>
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<tr>
<td>Sum of both effects (esmFixClim1 + esmFdbk1)</td>
<td>0.95</td>
<td>5.49 [5.09, 5.85]</td>
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<tr>
<td>380 – 535 ppm CO$_2$</td>
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<tr>
<td>Slow increase in CO$_2$ (RCP4.5)</td>
<td>0.93</td>
<td>2.84 [2.54, 3.08]</td>
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<td>Medium-fast increase in CO$_2$ (RCP8.5)</td>
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<td>2.38 [2.18, 2.55]</td>
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</tr>
<tr>
<td>Rapid increase in CO$_2$ (1pctCO2)</td>
<td>0.96</td>
<td>2.05 [1.94, 2.16]</td>
<td>–</td>
</tr>
</tbody>
</table>
Figure A1. Gedankenexperiment to examine the applicability of the Emergent Constraints analysis assuming an idealized linear behavior of the system (Case 1, Table A1). a, Changes in GPP relate linearly to changes in annual averaged atmospheric CO$_2$ concentration. The yellow band marks the projection period of interest, i.e. the period of concentration from $x + 4\Delta$ to $x + 5\Delta$. b, Changes in LAI relate linearly to changes in GPP. The parameterization in the linear functions for pseudo observations (dashed black-blue solid line) as well as models (solid grey lines) are determined randomly for each model. c, The diagnostic variable LAI sensitivity to, remains constant with increasing as a consequence of the overall linear characteristics of the system. The colored bands indicate three ‘past’ periods from $x$ to $x + \Delta$ area-weighted averaged GDD0 for NHL (blue), $x + \Delta$ to $x + 2\Delta$ (green solid line), and $x + 2\Delta$ to $x + 3\Delta$ their leading principal component $\omega$ (red). d, Linear relationships among the pseudo model ensembles (Ensemble LR 1-3 on top of each other, red) between LAI sensitivity to of the three past periods and $\Delta$GPP from the projected period. Red dots mark different models and the dashed line represents associated pseudo- observations for all three historical periods. Yellow solid line depicts the constant Emergent Constraint on projected $\Delta$GPP irrespective of the past period.
Figure A2. Similar pixel distribution of predictor and predictand in each model, except HadGEM2-ES. Histograms and associated probability density functions (Gaussian kernel density estimation) of LAI sensitivity to $\omega$ (red, left $y$-axis, historical simulations) and temporal trends in GPP (blue, right $y$-axis, 1pctCO2, until 2\times CO_2) for NHL are shown for all CMIP5 models. Only significant pixels are included (Mann-Kendall test, $p < 0.1$). To obtain comparability between the distributions, the $x$-axis was normalized and has only qualitative meaning.
Figure A3. Temporal variation of $LAI_{\text{max}}$ sensitivity to $\omega$ in four CMIP5 models analogous to Fig. 4. The colored lines show $LAI_{\text{max}}$ sensitivity variations for moving windows of varying length of 15 (blue), 30 (green), and 45 (red) years over the historical period from 1860 to 2005.
Figure A4. Correlation of $\Delta L_{AI,max}$ and $\Delta GPP$ with increasing CO$_2$ forcing, starting from a pre-industrial concentration of 280 ppm (1xCO$_2$) to 4xCO$_2$ (CMIP5 1pctCO2 simulations). Results are shown for four CMIP5 models analogous to Fig. 5. Blue colored dots show the relation between 1xCO$_2$ and 2xCO$_2$, green colored dots between 2xCO$_2$ and 3xCO$_2$, and red colored dots between 3xCO$_2$ and 4xCO$_2$. The respective colored lines represent the best linear fit through those dots and the shading represents the 95% confidence interval.
Figure A5. Thought experiment to examine the applicability of the EC analysis assuming an idealized linear / linear behavior of the system (Case 1, Table A1). a, Changes in GPP relate linearly to changes in CO₂ concentration. The yellow band marks the projection period of interest, i.e. the period of CO₂ concentration from \( x + 4\Delta \) to \( x + 5\Delta \). b, Changes in LAI relate linearly to changes in GPP. The parameterization in the linear functions for pseudo observations (dashed black line) as well as models (solid grey lines) are determined randomly for each model. c, The diagnostic variable, LAI sensitivity to CO₂, remains constant with increasing CO₂ as a consequence of the overall linear characteristics of the system. The colored bands indicate three 'past' periods from \( x \) to \( x + \Delta \) (blue), \( x + \Delta \) to \( x + 2\Delta \) (green), and \( x + 2\Delta \) to \( x + 3\Delta \) (red). d, Linear relationships among the pseudo model ensembles (Ensemble LR 1-3 on top of each other, red) between LAI sensitivity to CO₂ of the three past periods and \( \Delta \text{GPP} \) from the projected period. Red dots mark different models and the dashed line represents associated pseudo observations for all three historical periods. Yellow solid line depicts the constant EC on projected \( \Delta \text{GPP} \) irrespective of the past period.
Figure A6. *Gedankenexperiment—Thought experiment* to examine the applicability of the *Emergent Constraints—EC* analysis assuming an idealized nonlinear/non-linear behavior of the system (Case 4, Table A1). a, ΔGPP decreases with increasing CO₂ concentration (described by a hyperbolic tangent function). The yellow band marks the projected period of interest, i.e. the period of CO₂ concentration from $x + 4\Delta$ to $x + 5\Delta$. b, Also ΔLAI decreases with increasing GPP (described by a hyperbolic tangent function). The parameterization in the hyperbolic tangent functions for pseudo observations (dashed black line) as well as models (solid grey lines) are determined randomly for each model. c, The diagnostic variable, LAI sensitivity to CO₂, is decreasing with increasing CO₂ as a consequence of the overall saturating characteristics of the system. The colored bands indicate three 'past' periods from $x$ to $x + \Delta$ (blue), $x + \Delta$ to $x + 2\Delta$ (green), and $x + 2\Delta$ to $x + 3\Delta$ (red). d, Linear relationships among the pseudo model ensembles (Ensemble LR, colored lines) and the dashed lines represent associated pseudo observations for respective historical period. Colored dots mark different models and the yellow band indicates the three past periods. Yellow solid line depicts the constant *Emergent Constraint—EC* on projected ΔGPP irrespective of the past period.
1 **Table A1.** Overview of four possible cases of interaction between forcing, non-observable and observable identified in the Gedankenexperiment:

All linear, all nonlinear, and two mixed cases.

<table>
<thead>
<tr>
<th>Different assumptions</th>
<th>( \frac{d[\text{non-observable}]}{d[\text{forcing}]} ), e.g. ( \frac{d[\text{GPP}]}{d[\text{CO}_2]} )</th>
<th>( \frac{d[\text{observable}]}{d[\text{non-observable}]} ), e.g. ( \frac{d[\text{LAI}]}{d[\text{GPP}]} )</th>
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</thead>
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<tr>
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<td>linear</td>
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<td>nonlinear, nonlinear</td>
<td>linear</td>
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<td>linear</td>
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<tr>
<td>4</td>
<td>nonlinear, nonlinear</td>
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