

The multi-scale structure of the atmospheric energetic constraints on global-averaged precipitation

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Abstract

This study presents a multi-scale analysis of cross-correlations based on Haar fluctuations of global-averaged anomalies of precipitation (P), precipitable water vapor (PWV), surface temperature (T) and atmospheric radiative fluxes. The results revealed an

emergent transition between weak correlations at sub-yearly time-scales (down to ~5-days) to strong correlations at time-scales larger than about ~1-2 years (up to ~1-decade).

At multi-year time-scales, (i) Clausius-Clapeyron becomes the dominant control of PWV ($\rho_{PWV,T} \approx 0.9$); (ii) surface temperature averaged over global-land and over global-ocean (SST) become strongly correlated ($\rho_{T_{land},SST} \approx 0.6$); (iii) global-averaged precipitation

variability is dominated by energetic constraints - specifically the surface downwelling longwave radiative flux (DLR) ($\rho_{P,DLR} \approx -0.8$) displayed stronger correlations than the direct response to T fluctuations; (iv) cloud effects are negligible for the energetic constraints in (iii), which are dominated by clear-sky DLR. At sub-yearly time-scales, all correlations underlying these four results decrease abruptly towards negligible values.

Such a transition has important implications to understand and quantify the climate sensitivity of the global hydrological cycle. The validity of the derived correlation structure is demonstrated by reconstructing global precipitation time-series at 2-year resolution, relying on the emergent strong correlations (P vs clear-sky DLR). Such a simple linear sensitivity model was able to reproduce observed P anomaly time-series with similar accuracy to an (uncoupled) atmospheric model (ERA-20CM), and two climate reanalysis (ERA-20C and 20CR). The linear sensitivity breaks down at sub-yearly time-scales, where the underlying correlations become negligible. Finally, the relevance of the multi-scale framework and its potential for stochastic downscaling applications is demonstrated by deriving accurate monthly P probability density functions (PDFs) from

34 the reconstructed 2-year P time-series based on scale-invariant arguments alone. The
35 derived monthly PDFs outperforming the statistics simulated by ERA-20C, 20CR and
36 ERA-20CM in reproducing observations.

37

38 **1. Introduction**

39 The precipitation response to changes in increased concentrations of greenhouse gases is
40 a central topic for the climate science community. Although its regional variability is
41 essential to determine the societal impacts, global-averaged precipitation is an important
42 first-order climate indicator, and a measure of the global water cycle, that must be
43 accurately simulated if robust climate projections are to be obtained across a wide range
44 of spatial and temporal scales.

45 However, even the long-term response of global-averaged precipitation is still poorly
46 understood, constrained and simulated (Collins et al., 2013; Allan et al., 2014; Hegerl et
47 al., 2015), largely due to the limited knowledge on the complex interactions between the
48 key components of the atmospheric branch of the water cycle and its forcing mechanisms.
49 This problem is tackled here by employing a multi-scale analysis framework to study the
50 global-averaged precipitation variability, and its relation to two key governing
51 mechanisms: the Clausius-Clapeyron relationship and the constraints imposed by the
52 atmospheric energy balance.

53 The Clausius-Clapeyron relationship is a well-known mechanism controlling the
54 variability of the global water cycle. Assuming constant relative humidity, it implies that
55 fractional changes in global-averaged precipitable water vapor ($\Delta PWV/PWV$) are
56 linearly related to fluctuations of global-averaged near-surface air temperature (ΔT) (e.g.
57 Held & Soden, 2006; Schneider et al., 2010):

$$58 \frac{\Delta PWV}{PWV} \approx \alpha_{PWV,T} \Delta T, \quad (1)$$

59 where $\alpha_{PWV,T} \approx 0.07 \text{ K}^{-1}$ at temperatures typical of the lower troposphere. Numerous
60 studies have provided a robust confirmation for the Clausius-Clapeyron mechanism at
61 multi-decadal to centennial time-scales, while also reporting an analogous linear response
62 of global-averaged precipitation to surface temperature fluctuations (see e.g. Schneider et
63 al., 2010; Trenberth, 2011; O’Gorman et al., 2012; and Allan et al., 2014 for reviews). In
64 general, these previous investigations agree on the $\sim 7\%/K$ sensitivity coefficient for
65 precipitable water vapor. However, there is large spread on the global precipitation
66 sensitivity coefficient estimates, typically in the 1%/K to 3%/K range.

67 A widely recognized explanation for the **sub-Clausius-Clapeyron** sensitivity of
68 precipitation to temperature fluctuations at long temporal scales comes from the
69 atmospheric energy balance (Allen & Ingram, 2002; Stephens & Ellis, 2008; Stephens &
70 Hu, 2010). Specifically, averaging over the global atmosphere, the latent heat flux
71 associated with precipitation formation ($L_V P$, with P being the global-averaged
72 precipitation flux and L_V the latent heat of vaporization) should be in balance with the net
73 atmospheric radiative flux (R_{atm}) and the surface sensible flux (F_{SH}):

$$74 \quad L_V P + R_{atm} + F_{SH} \approx 0, \quad (2)$$

75 Equation (2) represents a general state of radiative convective equilibrium (Pauluis &
76 Held, 2002), with energy fluxes defined positive for atmospheric gain, and negative
77 otherwise.

78 If the **Clausius-Clapeyron** relationship was the dominant mechanism controlling the
79 response of atmospheric moisture content and the global water cycle to temperature
80 fluctuations, then global-averaged precipitable water vapor and precipitation could be
81 expected to be strongly correlated to surface temperature. Previously Gu and Adler (2011,
82 2012) found strong correlations between the inter-annual variability of global-averaged
83 precipitable water vapor and surface temperature, in tight agreement with the Clausius-
84 Clapeyron mechanism. However, they found weaker (yet significant) correlations
85 between the inter-annual variability of global-averaged precipitation and surface
86 temperature, raising doubts on whether the Clausius-Clapeyron mechanism could be
87 directly extendable to global precipitation. Notice, **however**, that these results focusing
88 on a single temporal scale might not represent the entire picture

89 A further source of complexity comes from the fact that precipitation and other relevant
90 atmospheric variables (including temperature, atmospheric moisture, wind, etc.) display
91 a complex statistical structure, with significant variability over a wide range of temporal
92 scales, and with the possibility of different mechanisms governing variability at different
93 time-scales (see e.g. Lovejoy & Schertzer, 2013 for a comprehensive review).
94 Furthermore, it has been shown that this complex multiscale structure plays a role (at
95 least) as important and the large amplitude periodic components, namely diurnal and
96 seasonal cycles (Lovejoy, 2015; Nogueira, 2017a). However, our understanding of the
97 underlying governing mechanisms at different time-scales remains largely elusive,
98 representing a central problem for future improvements to climate simulation and
99 projection.

100 Recently, Nogueira (2018) analyzed satellite-based observational datasets, a long Global
101 Climate Model (GCM) simulation and reanalysis products and found a tight correlation
102 (~ 0.8) between anomaly (deseasonalized) time-series of global-averaged precipitable
103 water vapor and surface temperature, which emerged at time-scales larger than ~ 1 -2
104 years. In contrast, at smaller time-scales the correlation decreased rapidly towards
105 negligible values (< 0.3). In other words, the **Clausius-Clapeyron** relationship is the
106 dominant mechanism of atmospheric moisture anomalies at multi-year time-scales, but
107 not at sub-yearly time-scales. Nogueira (2018) also found that the magnitude of the
108 correlations between anomaly time-series for global-averaged precipitation and surface
109 temperature was negligible at sub-yearly time-scales, while at multi-year time-scales the
110 results showed large spread amongst different data-sets, ranging between negligible
111 (< 0.3) and strong (~ 0.8) correlation values. Building on this previous study, here the
112 multi-scale analysis of the mechanisms governing global precipitation variability was
113 extended, including the energetic constraints on precipitation represented in Eq. (2). **The**
114 **manuscript is organized as follows: section 2 describes the considered datasets and the**
115 **multi-scale analysis framework; the results of multi-scale correlation analysis on**
116 **precipitation variability are presented and discussed in section 3; in section 4 the validity**
117 **of the linear sensitivity correlations derived from the multi-scales analysis is**
118 **demonstrated by employing a simple linear model to reconstruct global-averaged**
119 **precipitation time-series from energetic constraints. At sub-yearly time-scales, where the**
120 **correlations break down, it is shown in section 5 how the monthly statistics can be**
121 **reproduced by employing a stochastic downscaling algorithm based on scale-invariant**
122 **symmetries of precipitation. Finally, the main conclusions are summarized and discussed**
123 **in section 6.**

124

125 **2. Data and Methodology**

126 **2.1. Data sets**

127 Precipitation observations were obtained from the Global Precipitation Climatology
128 Project (GPCP) version 2.3 monthly precipitation dataset (Adler et al., 2003), which
129 covers the full globe at 2.5° resolution from 1979 to present. Gridded datasets of monthly
130 average surface temperatures were obtained from the Goddard Institute for Space Studies
131 (GISSTEMP) analysis (Hansen et al., 2010), which covers the globe at 2° resolution from
132 1880 to present, with the values provided as anomalies relative to the 1951-1980 reference
133 period. GISSTEMP blends near-surface air temperature measurements from

134 meteorological stations (including Antarctic stations) with a reconstructed sea surface
135 temperature (SST) dataset over oceans. Observations of atmospheric radiative fluxes
136 were obtained from the National Aeronautics and Space Administration (NASA) Clouds
137 and the Earth's Radiant Energy System, Energy Balanced and Filled (CERES-EBAF)
138 Edition 4.0 (Loeb et al., 2009), a monthly dataset covering the full globe at 1° resolution
139 from March/2000 to June/2017.

140 Two state-of-the-art reanalyses of the twentieth-century were considered in the present
141 study. One was the National Oceanic and Atmospheric Administration Cooperative
142 institute for Research in Environmental Sciences (NOAA-CIRES) twentieth-century
143 reanalysis (20CR) version 2c (Compo et al., 2011), which covers the full globe at 2°
144 resolution, spanning from 1851 to 2014. Only surface pressure observations and reports
145 are assimilated in this reanalysis. SST boundary conditions are obtained from 18 members
146 of pentad Simple Ocean Data Assimilation with Sparse Input (SODAsi) version 2, with
147 the high latitudes corrected to the Centennial in Situ Observation-Based Estimates of the
148 Variability of SST and Marine Meteorological Variables, version 2 (COBE-SST2). Here,
149 global-mean time-series of precipitation, precipitable water vapor, near-surface
150 temperature, SST, and atmospheric radiative fluxes were obtained from 20CR at daily
151 resolution for the 1900-2010 period. Notice that the net atmospheric radiative flux cannot
152 be obtained from 20CR, because the incoming solar radiation at the top of the atmosphere
153 is not available for this dataset, due to an error with output processing.

154 The other reanalysis considered in the present study was the European Centre for
155 Medium-Range Weather Forecasts (ECMWF) twentieth-century reanalysis (ERA-20C,
156 Poli et al., 2015), which covers the full globe at 1° resolution spanning from 1900-2010.
157 It assimilates marine surface winds from the International Comprehensive Ocean-
158 Atmosphere Data Set version 2.5.1 (ICOADSv2.5.1) and surface and mean-sea-level
159 pressure from the International Surface Pressure Databank version 3.2.6 (ISPDv3.2.6)
160 and from ICOADSv2.5.1. SST boundary conditions are obtained from the Hadley Centre
161 Sea Ice and Sea Surface Temperature data set version 2.1 (HadISST2.1). Global-mean
162 time-series of precipitation, precipitable water vapor, near-surface temperature, SST, and
163 atmospheric radiative fluxes were obtained from ERA-20C at daily resolution for the
164 1900-2010 period.

165 Finally, the uncoupled ECMWF twentieth-century ensemble of ten atmospheric model
166 integrations (ERA-20CM, Hersbach et al., 2015) was considered, which uses the same
167 model, grid, initial conditions, radiative and aerosol forcings as ERA-20C. However, no

168 observations are assimilated, the simulation is integrated continuously over the full 1900-
 169 2010 period, and SST is prescribed by an ensemble of realizations from HadISST2.1,
 170 including one control simulation and nine simulations with perturbed SST and sea-ice
 171 concentration. A 10-member ensemble of global-mean time-series of precipitation,
 172 precipitable water vapor, near-surface temperature, SST, and atmospheric radiative fluxes
 173 were obtained from ERA-20CM at monthly resolution for the 1900-2010 period.
 174 Considering ERA-20CM allowed to test the sensitivity of the multi-scale correlation
 175 structure derived from ERA-20C to data assimilation, but different atmospheric
 176 evolutions associated with perturbations to the forcing fields (particularly to SST).
 177 Notice that the clear-sky radiative fluxes considered here obtained from ECMWF datasets
 178 are computed for the same atmospheric conditions of temperature, humidity, ozone, trace
 179 gases and aerosol, but assuming that the clouds are not there. Clear-sky estimates from
 180 ERA-20C and ERA-20CM cover the full globe area and not just the cloud free regions at
 181 each time instant. However, they are available for net radiative fluxes, but not for
 182 downwelling or upwelling radiation fluxes.

183 2.2. Multi-scale correlation analysis

184 Consider two time-series, y , and y' , with N data points each. Here the goal is to study the
 185 correlation between the fluctuations $\Delta y(\Delta t)$ and $\Delta y'(\Delta t)$ at different time-scales Δt . Due
 186 to the strong yearly cycle present in the considered time-series, the periodic seasonal trend
 187 was first eliminated by subtracting the long-term average (over all the years in the record)
 188 of each calendar day (or month, depending on temporal resolution):

$$189 y_{ds}(i) = y(i) - \langle y \rangle_d, \quad (3)$$

190 where y_{ds} is the deseasonalized anomalies time-series.

191 Traditionally, fluctuations are defined by the difference $\Delta y(\Delta t) = y(t + \Delta t) - y(t)$.
 192 However, it has been shown that such definition is only appropriate for fluctuations
 193 increasing with time-scale (Lovejoy and Schertzer, 2013). Consequently, the traditional
 194 definition is not useful for the present study, since the fluctuations for most atmospheric
 195 variables time-series (including temperature, rain, wind, water vapor, amongst others)
 196 decrease with increasing time-scale over the tens of days to tens of years range (e.g.
 197 Lovejoy and Schertzer, 2013; Lovejoy, 2015; Lovejoy et al., 2017; Nogueira, 2017a;
 198 2017b; 2018). In this sense, here the fluctuations were defined using the Haar wavelet,
 199 which is appropriate for the full range of time-scales and all atmospheric variables
 200 considered, in both cases where fluctuations increase or decrease with time-scale.
 201 Furthermore, correlations computed from Haar fluctuation time-series also avoid the low

202 frequency biases that emerge in standard correlation analysis due to climate variability
203 (see Lovejoy et al. (2017) for a detailed description of the Haar fluctuations and
204 correlations of Haar fluctuations).

205 The Haar fluctuations are simply defined as the difference of the means from $t + \Delta t/2$ to
206 $t + \Delta t/2$ and from t to $t + \Delta t/2$, i.e.:

$$207 (\Delta y(\Delta t))_{Haar} = \frac{2}{\Delta t} \int_{t+\Delta t/2}^{t+\Delta t} y(t) dt - \frac{2}{\Delta t} \int_t^{t+\Delta t/2} y(t) dt, \quad (4)$$

208 For the sake of simplicity, henceforth the fluctuation notation $\Delta y(\Delta t)$ will be employed
209 to refer to Haar fluctuations (i.e. $\Delta y(\Delta t) \equiv (\Delta y(\Delta t))_{Haar}$). A Haar fluctuation time-
210 series was computed by employing Equation 4 at each instant of the deseasonalized
211 anomalies time-series for each variable considered. Finally, at each time-scale, Δt , the
212 correlation coefficient, ρ , of the corresponding Haar fluctuations time-series was
213 computed for each pair of variables considered.

214 Notice that, in computing correlations at time-scales larger than two times the original
215 time-series resolution, there is overlapping of the data-points considered in computing the
216 Haar fluctuations. While this could introduce spurious effects in the computed
217 correlations, previous works have shown the robustness of the Haar fluctuation-based
218 correlations methodology used here (e.g. Lovejoy et al., 2017). Additionally, the
219 analogous method of Detrended Cross-Correlation Analysis has also been employed on
220 overlapping windows and demonstrated to provide accurate correlation estimates at
221 different time-scales using overlapping windows (see e.g. Podobnik & Stanley, 2008;
222 Podobnik et al., 2011; Piao and Fu, 2016). In fact, in Section 3 below it is shown that
223 identical correlation structures are obtained between correlations of Haar fluctuations and
224 Detrended Cross-Correlation Analysis. Since the multi-scale cross-correlation structure
225 obtained with Haar fluctuations is identical to the results using Detrended Cross-
226 Correlations Analysis, it is assumed that critical points for the 95% significance level of
227 Haar fluctuation correlations are identical to the ones demonstrated by Podobnik et al.
228 (2011) for Detrended Cross-Correlation Analysis using overlapping windows, where the
229 significant values can vary between values below 0.1 and up to about 0.4, depending on
230 the time series length, the considered time-scale, and the power law exponents of both
231 time-series. In this sense, here it is assumed that correlation magnitudes below 0.3 are
232 nonsignificant, and that magnitudes in the 0.3 to 0.4 range should be interpreted with care.

233

234 3. Analysis of the mechanisms governing P variability across time-scales

235 **3.1. Multi-scale structure of the atmospheric water cycle response to surface**
236 **temperature fluctuations**

237 The correlations between Haar fluctuations time-series revealed strong correlations (~ 0.9)
238 between deseasonalized anomaly time-series for global-averaged precipitable water
239 vapor and near surface temperature (or, alternatively, SST) at multi-year time-scales (Fig.
240 1a). However, as the time-scale decreases there is a transition in the correlation structure,
241 and negligible correlations (< 0.3) emerge at sub-yearly time-scales. This result suggested
242 that the Clausius-Clapeyron relationship (see Eq. (1)) holds to a very good approximation
243 at multi-year time-scales, but not at sub-yearly time-scales. Interestingly, Lovejoy et al.
244 (2017) computed the Haar fluctuation correlations for GISSTEMP surface temperatures
245 and found a similar transition in the multi-scale correlation structure of SST against
246 global-averaged surface temperature, with low-correlations at time-scales below a few
247 months and strong correlations (~ 0.8) at multi-year time-scales. Notice that the latter
248 strong correlations weren't surprising, since SST was a major component in their
249 definition of global-averaged surface temperature (which for GISSTEMP uses SST over
250 the ocean pixels and 2-meter air temperature over land pixels). Nonetheless, Lovejoy et
251 al. (2017) also found a similar transition for the correlation between SST and near-surface
252 air temperature averaged over global-land, with maximum correlation values ~ 0.6 at
253 multi-year time-scales. The transition in the correlation structure between SST and
254 global-land temperature was confirmed here for ERA-20C, ERA-20CM, 20CR and
255 GISSTEMP (Fig. 1b). Thus, the present results support Lovejoy et al. (2017) argument
256 that these abrupt correlation changes correspond to a fundamental behavioral transition,
257 where the atmosphere and the oceans start to act as a single coupled system. Furthermore,
258 the results presented here suggest that precipitable water vapor anomalies at multi-year
259 resolution can be derived, to a very good approximation, from SST alone.

260 Nogueira (2018) also reported a transition in the multi-scale correlation structure between
261 deseasonalized anomaly time-series of global-averaged precipitation and surface
262 temperature (considering SST over the oceans and 2-m air temperature over land), with
263 negligible values at sub-yearly time-scales, but with large spread in the magnitude of the
264 multi-year correlations, ranging between ~ 0.3 and ~ 0.8 . Here, a similar result was found
265 for the multi-scale correlations structure between global-averaged precipitation and
266 surface temperature and, also, global-averaged precipitation and SST (Fig. 1c), with large
267 spread in correlation magnitude at multi-year time-scales (~ 0.7 in ERA-20C and ERA-
268 20CM, ~ 0.6 in 20CR, and < 0.4 in observations). Furthermore, considering different time-

269 lags in computing the cross-correlations between precipitation and surface temperature
270 did not reveal the presence of significant lagged correlations over the daily to decadal
271 time-scale range.

272

273 **3.2. Multi-scales structure of the energetic constraints to precipitation variability**

274 A study of the circulation component of the precipitation response to temperature
275 fluctuations requires a detailed representation of several spatially heterogeneous variables
276 and their nonlinear interactions. An alternative path towards understanding global-
277 averaged precipitation temporal variability was taken in the present investigation,
278 focusing on the constraints imposed by the atmospheric energy balance represented in
279 Equation (2). Fig. 2a (solid lines) shows that the estimated multi-scale correlation
280 coefficients between the deseasonalized anomaly time-series for precipitation and net
281 atmospheric radiative fluxes were strongly (negatively) correlated at multi-year time-
282 scales ($\rho \sim -0.8$ in ERA-20C, ERA-20CM and observations), in agreement with the
283 balance in Equation (2). In contrast, at sub-yearly time-scales the correlation magnitude
284 decreased rapidly, changed sign around monthly time-scales, and reached values ~ 0.4 at
285 time-scales below about 10 days.

286 Considering the combined effect of the net atmospheric radiative fluxes and sensible heat
287 flux in Equation (2) slightly increased the (positive) correlations at sub-monthly time-
288 scales (Fig. 2a, dashed lines), although the absolute changes are essentially below 0.1.
289 More importantly, Fig. 2a shows that the magnitude of the correlation at multi-year time-
290 scales between global-averaged precipitation and net atmospheric radiative fluxes is
291 significantly larger than when the combined effect of net atmospheric radiative fluxes and
292 sensible heat flux were considered. Indeed, the correlation between global-averaged
293 precipitation and sensible heat flux displayed values up to about 0.5 at sub-monthly time-
294 scales, but essentially < 0.4 at multi-year time-scales (Fig. 2a, dot-dashed lines). Given
295 the results in Fig. 2a, the following linear relation was hypothesized: $L_V \Delta P \approx$
296 $c_1 \times (-\Delta R_{atm}) + c_2$, where c_1 and c_2 are arbitrary constants, and Δ represents
297 fluctuations taken as deseasonalized anomalies at multi-year resolutions. At sub-yearly
298 time-scales this simplification is not adequate, since the correlations between global-
299 averaged precipitation and net atmospheric radiative fluxes becomes negligible. In other
300 words, the energy balance represented in Equation (2) doesn't represent the dominant
301 constraint on precipitation variability at sub-yearly time-scales, most likely due to non-
302 negligible changes in atmospheric heat storage.

303 The analysis was extended by decomposing net atmospheric radiative fluxes into its net
 304 atmospheric longwave and shortwave radiative flux components, i.e. $R_{atm} = R_{LW,net} +$
 305 $R_{SW,net}$. On the one hand, the correlation between global-averaged precipitation and net
 306 atmospheric radiative fluxes is nearly identical to the correlation between global-averaged
 307 precipitation and net atmospheric longwave radiative fluxes (i.e. $\rho_{P,R_{atm}} \approx \rho_{P,R_{LW,net}}$)
 308 over the full range of time-scales considered (Fig. 2b). On the other hand, $\rho_{P,R_{SW,net}}$ also
 309 displayed significant values (~ 0.6) at multi-year time-scales, but the latter magnitude was
 310 nearly 0.2 lower when compared to $\rho_{P,R_{atm}}$ and $\rho_{P,R_{LW,net}}$ (Fig. 2b). Consequently, the
 311 above linear relationship for multi-scale P anomalies was further refined as $L_V \Delta P \approx$
 312 $c_1 \times (-\Delta R_{atm}) + c_2 \approx c_3 \times (-\Delta R_{LW,net}) + c_4$, where c_3 and c_4 are arbitrary constants.
 313 Subsequently, the net atmospheric longwave radiative flux was further decomposed into
 314 the top-of-atmosphere (TOA) and surface net longwave fluxes, i.e. $R_{LW,net} = R_{LW,TOA} +$
 315 $R_{LW,SFC}$. At multi-year time-scales, $\rho_{P,R_{atm}} \approx \rho_{P,R_{LW,SFC}}$ (Fig. 2c), suggesting that the
 316 surface net longwave radiative fluxes provide the main constraint to global-averaged
 317 precipitation variability. The correlation between global-averaged precipitation and TOA
 318 longwave radiative fluxes also displayed significant values at multi-year time-scales, up
 319 to ~ -0.6 in ERA-20C and ERA-20CM datasets, but much lower in 20CR where the
 320 magnitude of the correlation was < 0.4 at multi-year time-scales. Nonetheless, the former
 321 correlations (in ERA-20C and ERA-20CM) were in better agreement with observations,
 322 suggesting that significant (negative) correlations existed between global-averaged
 323 precipitation and net longwave fluxes at TOA anomalies at multi-year time-scales.
 324 However, for all datasets, the magnitude of $\rho_{P,R_{LW,TOA}}$ at multi-year time-scales was
 325 nearly 0.2 lower than for $\rho_{P,R_{LW,SFC}}$. Consequently, a further approximation was
 326 considered on the linear model for precipitation fluctuations at multi-year time-scales:
 327 $L_V \Delta P \approx c_1 \times (-\Delta R_{atm}) + c_2 \approx c_3 \times (-\Delta R_{LW,net}) + c_4 \approx c_5 \times (-\Delta R_{LW,SFC}) + c_6$.
 328 Finally, the surface net longwave radiative flux can be further decomposed into its
 329 upwelling and downwelling (henceforth denoted downwelling longwave radiation, DLR)
 330 components. Fig. 2d shows that, at multi-year time-scales, the differences in the
 331 correlations of global-averaged precipitation against DLR ($\rho_{P,DLR}$) or against net
 332 atmospheric radiative fluxes (i.e. $\rho_{P,R_{atm}}$) were within 0.1 in observations, ERA-20C and
 333 ERA-20CM (R_{atm} is unavailable for 20CR). Thus, at multi-year time-scales, the
 334 fluctuations in downwelling surface longwave radiative fluxes are, to a good

335 approximation, linearly related to precipitation fluctuations: $L_V \Delta P \approx c_7 \times (-\Delta DLR) +$
336 c_8 . Notice that the correlation structure of global-averaged precipitation against
337 upwelling surface radiative fluxes or against net atmospheric radiative fluxes are nearly
338 identical in observations. However, significant difference emerged between these two
339 quantities (~ 0.2) in ERA-20CM and ERA-20C. Thus, a similar linear relationship
340 between ΔP and $\Delta R_{LW,SFC,UP}$ might also hold to a good approximation, although the
341 results are less robust than for ΔP against ΔDLR .

342 The correlation between global-averaged precipitation and clear-sky net radiative
343 atmospheric heating (i.e. $\rho_{P,R_{atm,cs}}$) was nearly identical to $\rho_{P,R_{atm}}$ at multi-year time-
344 scales (Fig. 3a). This suggested that the cloud effects on the multi-year linear dependence
345 between precipitation variability and net atmospheric radiative fluxes may be neglected.
346 But the same isn't true at time-scales below a few months, where significant differences
347 emerge between $\rho_{P,R_{atm,cs}}$ and $\rho_{P,R_{atm}}$. The clear-sky approximation holds at multi-year
348 time-scales for correlations of global-averaged precipitation against net atmospheric
349 longwave radiative fluxes and, also, and against the global-averaged net surface longwave
350 fluxes (Fig. 3b). Based on these results, it was further hypothesized that cloud effects are
351 also negligible for the correlation between global-averaged precipitation and DLR at
352 multi-year temporal scales. This hypothesis could not be tested directly because clear-sky
353 DLR time-series were not available for the ECMWF datasets. Nonetheless, the results in
354 Section 4 based on an empirical algorithm for DLR estimation under a clear-sky
355 approximation provided support for this hypothesis.

356 At this point, it is important to notice that the existence of strong correlations does not
357 necessarily imply causality between two variables. However, the atmospheric energy
358 balance in Equation (2) provides a physical basis for the obtained strong (negative)
359 correlations values between precipitation and atmospheric radiative fluxes. In fact, the
360 multi-scale analysis presented here provided further robustness to previous investigations
361 on the importance of energetic constraints to global precipitation, the dominant role of
362 surface longwave fluxes, namely DLR, and the negligible cloud effects in these
363 relationships (e.g., Stephens and Hu, 2010; Stephens et al., 2012a,b). More importantly,
364 a clear transition emerged between robust correlations at multi-year time-scales and
365 negligible correlations at sub-yearly time-scales, which was found for global-averaged
366 precipitation against atmospheric radiative fluxes (particularly total net, net longwave and
367 DLR), global-averaged precipitable water vapor against surface temperature (and SST),

368 for global SST against global near-surface air temperature and, less robustly, for global-
369 averaged precipitation against surface temperature (or SST).

370 Notice that the correlation structure derived from Haar fluctuations of different
371 combinations of variables presented in the present section are identical to the correlation
372 structure obtained by employing Detrended Cross-Correlation Analysis (DCCA, see
373 Supplementary Figures 1, 2 and 3). DCCA has been previously shown to robustly
374 quantify the correlations at different time-scales (Podobnik & Stanley, 2008; Piao and Fu,
375 2016; Nogueira, 2017b; 2018, where detailed descriptions of DCCA methodology are
376 also provided). This result provides one of the first empirical verifications for the multi-
377 scale correlation obtained from Haar fluctuations, recently introduced by Lovejoy et al.
378 (2017).

379

380 **4. Evaluation of the multi-year linear relationships between global-averaged** 381 **precipitation and clear-sky DLR and surface temperature**

382 The strong correlations between global-averaged precipitation and atmospheric longwave
383 radiative fluxes imply that simple linear model should be able to reproduce the variability
384 precipitation anomalies at multi-year time-scales. This hypothesis is tested in the present
385 section, aiming to provide robustness to the strong multi-year correlations presented in
386 Section 3. Specifically, the robustness of the tight correlation between global-averaged
387 precipitation and clear-sky DLR at multi-year time-scales is tested. Additionally, it is
388 tested whether the more robust correlation between global-averaged precipitation and
389 clear-sky DLR at multi-year time-scales compared to global-averaged precipitation
390 against surface temperature results in a better reconstruction of precipitation variability
391 by such a linear model.

392 The clear-sky DLR can be derived, to a good approximation, from the global averaged
393 near-surface temperature alone (e.g. Stephens et al., 2012b). Furthermore, given the tight
394 coupling between global-averaged temperature over land and SST at multi-year time-
395 scales (Fig. 1b), it is hypothesized that clear-sky DLR variability could be obtained, to a
396 good approximation directly from the SST forcing. This hypothesis is also supported by
397 the nearly identical correlations between global-averaged precipitable water vapor against
398 surface temperature or against SST (Fig. 1a).

399 Here two different algorithms to estimate clear-sky DLR are tested: the Dilley-O'Brien
400 model (Dilley & O'Brien, 1998), and the Prata model (Prata, 1996). In the Dilley-O'Brien
401 model:

402
$$DLR_{2y,DO} = a_1 + a_2 \left(\frac{SST_{2y}}{SST_c} \right)^6 + a_3 \left(\frac{\Delta PWV_{2y} + PWV_c}{PWV_c} \right)^{1/2}, \quad (8)$$

403 Where $a_1 = 59.38 \text{ Wm}^{-2}$, $a_2 = 113.7 \text{ Wm}^{-2}$ and $a_3 = 96.96 \text{ Wm}^{-2}$ are the model parameters,
 404 and $PWV_c = 22.5 \text{ kg m}^{-2}$ is the climatological value for precipitable water vapor. The
 405 subscript ‘2y’ (e.g. DLR_{2y}) indicates a fluctuation for $\Delta t = 2$ -year. Notice that $DLR_{c,DO} =$
 406 $a_1 + a_2 + a_3$ is obtained by replacing the climatological values of PWV and SST in
 407 Equation (8).

408 The Prata model for $\Delta DLR_{2y,Pr}$ is based on the Stefan-Boltzmann equation:

409
$$DLR_{2y,Pr} = \varepsilon_{clr} \sigma_{SB} SST_{2y}^4 \quad (9)$$

410 Where $\sigma_{SB} = 5.67 \times 10^{-8} \text{ Wm}^{-2}\text{K}^{-4}$ is the Stefan-Boltzmann constant and:

411
$$\varepsilon_{clr} = 1 - \left(1 + PWV_{2y} \right) \exp \left(- \left(1.2 + 3PWV_{2y} \right)^{1/2} \right) \quad (10)$$

412 The anomaly-time series is computed from $\Delta DLR_{2y,Pr} = DLR_{2y,Pr} - DLR_{c,Pr}$, where
 413 $DLR_{c,Pr}$ is obtained by replacing the climatological values of PWV and SST in Equations
 414 (9) and (10).

415 The strong correlation between global-averaged precipitable water vapor and SST at
 416 multi-year time-scales (Fig. 1a) allowed to remove the PWV dependence in Equations (8)
 417 and (11), by replacing $PWV_{2y} \approx \alpha_{PWV,SST} \Delta SST_{2y} + PWV_c$. The forcing ΔSST_{2y} time-
 418 series were obtained by coarse-graining the deseasonalized (using Equation (3)) global-
 419 averaged SST obtained from GISSTEMP dataset. The sensitivity coefficient, $\alpha_{W,SST} \approx$
 420 0.08 K^{-1} was estimated by least-square regression of $\Delta PWV_{2y}/PWV_c$ against ΔSST_{2y} ,
 421 pooling together all datasets (ERA-20C, ERA-20CM and 20CR). The $\alpha_{PWV,SST}$ estimates
 422 are summarized in Table 1, including for each individual dataset, ranging between 0.07
 423 and 0.10 K^{-1} . Notice that the obtained values are close to the typical 0.07 K^{-1} value
 424 predicted by the Clausius-Clapeyron relationship.

425 In this way, two reconstructed anomaly time-series for global-averaged precipitation were
 426 obtained using the Diley-O’Brien and the Prata algorithms. The climatological global-
 427 averaged precipitation $P_c \approx 2.7 \text{ mm/day}$ was estimated from GPCP dataset. The
 428 sensitivity coefficient $\alpha_{P,DLR} \approx 0.004 \text{ (W/m}^2\text{)}^{-1}$ was estimated by least-square regression
 429 of $\Delta P_{2y}/P_c$ against ΔDLR_{2y} , pooling together all available datasets (ERA-20C, ERA-
 430 20CM, 20CR and GPCP against CERES-EBAF). Notice that, in estimating $\alpha_{P,DLR}$, clear-
 431 sky DLR time-series were used where available (i.e. for ERA-20C and ERA-20CM)
 432 datasets, but they were replaced by (full-sky) DLR otherwise (i.e. for 20CR and CERES-
 433 EBAF). The $\alpha_{P,DLR}$ estimates are summarized in Table 2, including values obtained from

434 each dataset (no estimate was made for GPCP against CERES-EBAF due to the limited
435 duration of the latter), ranging between $0.003 \text{ (W/m}^2\text{)}^{-1}$ and $0.005 \text{ (W/m}^2\text{)}^{-1}$.

436 **Another simple linear model for reconstruction of multi-year global-averaged**
437 **precipitation** anomaly time-series was tested, based on the direct response (correlations)
438 of P to SST fluctuations, i.e. $P_{2y,SST} \approx \alpha_{P,SST} \Delta SST_{2y} P_c + P_c$. Again, the ΔSST_{2y} was
439 obtained from GISSTEMP dataset. The sensitivity coefficient, $\alpha_{P,SST} \approx 0.02 \text{ K}^{-1}$ was
440 estimated by least-square regression of $\Delta P_{2y}/P_c$ against ΔSST_{2y} , pooling together all
441 datasets (ERA-20C, ERA-20CM, 20CR and GPCP against GISSTEMP). The $\alpha_{P,SST}$
442 estimates are summarized in Table 3, including for each individual dataset, ranging
443 between 0.02 and 0.04 K^{-1} . Notice that the obtained values are close to the 0.01 to 0.03
444 K^{-1} range reported in the relevant literature (e.g. Schneider et al., 2010; Trenberth, 2011;
445 O’Gorman et al., 2012; and Allan et al., 2014).

446 When compared against ΔP_{2y} directly derived from GPCP for the 1979 to 2010 period,
447 the errors in the proposed linear ΔP_{2y} reconstructions were generally close to those for
448 atmospheric model-based products (Fig. 4). $\Delta P_{2y,Pr}$ displays the highest mean bias,
449 somewhat higher than for atmospheric model-based datasets, but also higher than the
450 mean bias for $\Delta P_{2y,DO}$ and $\Delta P_{2y,SST}$ (Fig. 4a). Notice that all atmospheric model-based
451 products considered here also display a positive bias. While this may be due a negative
452 bias of GPCP (e.g. Gehne et al., 2015), this observational dataset represents the longest
453 reliable dataset for global precipitation studies and thus was considered here as ‘the truth’.
454 More importantly, the mean bias is easy to correct, simply by subtracting its value from
455 the time-series. This correction was implemented here for all atmospheric model-based
456 and linear-model based ΔP_{2y} time-series. Figure 4c shows that the normalized standard
457 deviation ($\sigma_n = \sigma_{2y,model}/\sigma_{2y,obs}$) estimated from $\Delta P_{2y,DO}$ (~ 0.4) and, particularly, from
458 $\Delta P_{2y,SST}$ (~ 0.3) were lower than the values estimated from atmospheric model-based
459 products (~ 0.5 - 0.9). In contrast, σ_n estimated from $\Delta P_{2y,Pr}$ was nearly 0.8 , which was
460 higher than 20CR and most members in the ERA-20CM ensemble, only outperformed by
461 ERA-20C dataset. The root-mean squared error after bias-correction (RMSE_{bc}) estimated
462 from $\Delta P_{2y,Pr}$ and $\Delta P_{2y,DO}$ were well within the range of the values obtained from
463 atmospheric model-based products (Fig. 4b), with the Prata model slightly
464 overperforming the Dilley-O’Brien model. RMSE_{bc} estimated from $\Delta P_{2y,SST}$ was on the
465 high-end of the atmospheric model-based range of values, and somewhat worse than for

466 the DLR-based linear models. Finally, the Pearson correlation coefficient between models
467 and observations (Fig. 4d) was similar amongst all linear-based models and well within
468 the range of values estimated from the atmospheric model-based products. The latter
469 result was expected since all linear models were forced by the same SST time-series.
470 Overall, these results suggested that $\Delta P_{2y,Pr}$ (after bias correction) reproduced the
471 observations with similar accuracy to atmospheric model-based products, including
472 similar $RMSE_{bc}$, variability amplitude and phase of the signal. $\Delta P_{2y,DO}$ displayed similar
473 performance for $RMSE_{bc}$ and for the phase, but not for the variability amplitude. Finally,
474 $\Delta P_{2y,SST}$ had the worst performance concerning $RMSE_{bc}$, but also in capturing the
475 variability amplitude, while it displayed similar ability to the other linear models in
476 reproducing the phase. The overall weakest performance of $\Delta P_{2y,SST}$ was coherent with
477 the less robust correlations underlying this model. Additionally, the results indicate that
478 the non-linear transformations on SST employed in the Prata and the Dilley-O'Brien
479 algorithms improved the linear models.

480

481 **5. Exploring scale-invariance for stochastic downscaling of precipitation to** 482 **monthly resolution**

483 At sub-yearly time-scales, the magnitude of the correlations between global-averaged
484 precipitable water vapor and SST, precipitation and DLR, and precipitation and
485 SST decreases abruptly to negligible values (cf. Section 3). Additionally, the cloud-
486 effects on the energetic constraints of precipitation variability become non-negligible
487 (Fig. 3). Consequently, the linear relationships underlying the above simple linear
488 reconstructions of global-averaged precipitation at 2-year resolution are no longer
489 appropriate at sub-yearly time-scales. Previous investigations have suggested that this
490 transition should be related to a fundamental transition in the stochastic scale-invariant
491 behavior of climate variables, which separates a high-frequency weather regime that
492 extends up to about the synoptic scales (around 10 days to 1-month in the atmosphere,
493 and around 1-year in the oceans) from a low-frequency weather (or macroweather) regime
494 that extends up to a few decades (see e.g. Lovejoy et al., 2017; Nogueira, 2018). In the
495 weather regime the amplitude of the fluctuations tends to increase with time-scale, while
496 in the macroweather regime the amplitude of the fluctuations tends to decrease with
497 increasing time-scale, hence implying a convergence toward the 'climate normal' at time-
498 scales of a few decades (Lovejoy, 2015).

499 In the present section, it is shown that the multi-scale analysis framework can also be
 500 taken advantage to perform stochastic downscaling from the multi-year to monthly
 501 resolution. This exercise allows to demonstrate the relevance of understanding and
 502 characterizing the multi-scale structure of atmospheric variables and its remarkable
 503 potential for stochastic downscaling applications.

504 Building on the strong scale-invariant symmetries present in the variability of global and
 505 regional precipitation across wide ranges of time-scales (e.g. Lovejoy and Schertzer,
 506 2013; Nogueira et al., 2013; Nogueira and Barros, 2014, 2015; Nogueira, 2017, 2018), an
 507 algorithm was proposed here to derive the sub-yearly statistics from the multi-year
 508 information alone. The physical basis for this algorithm is that while the atmosphere is
 509 governed by continuum mechanics and thermodynamics, it simultaneously obeys
 510 statistical turbulence cascade laws (e.g., Lovejoy & Schertzer, 2013; Lovejoy et al.,
 511 2018).

512 Conveniently, precipitation (and many other atmospheric variables) is characterized by
 513 low spectral slopes $\beta < 1$, with quasi-Gaussian and quasi-non-intermittent statistics, at
 514 time-scales between ~ 10 days and a few decades (Lovejoy & Schertzer, 2013; de Lima
 515 & Lovejoy, 2015; Lovejoy et al., 2015, 2017; Nogueira, 2017b, 2018). Grounded by these
 516 scale-invariant properties, fractional Gaussian noise was used here to generate multiple
 517 realizations of downscaled ΔP at monthly resolution from each member of each ΔP_{2y}
 518 time-series:

$$519 \quad \Delta P_{1m}(t) = fGn_{1m}(t) \frac{\Delta P_{2y}(t)}{fGn_{2y}(t)} \quad (11)$$

520 where fGn_{1m} is a fractional Gaussian noise, which was computed by first generating a
 521 random Gaussian noise (g), then taking its Fourier transform (\tilde{g}), multiplying by $k^{-\beta/2}$,
 522 and finally taking the inverse transform to obtain fGn_{1m} . The mean of fGn_{1m} was forced
 523 to be equal to the number of data-points of ΔP_{2y} . Then fGn_{2y} was obtained by coarse-
 524 graining fGn_{1m} using 24-point (i.e. 2 years) length boxes. In this way, $\Delta P_{1m,DO}$, $\Delta P_{1m,Pr}$,
 525 $\Delta P_{1m,SST}$ ensembles are generated respectively from the bias-corrected $\Delta P_{2y,DO}$, $\Delta P_{2y,Pr}$
 526 and $\Delta P_{2y,SST}$ time-series. One hundred plausible realizations are generated for each
 527 ensemble, corresponding to one hundred different realizations of fGn_{1m} . Based on recent
 528 investigations on P scale-invariance properties, a spectral exponent $\beta \approx 0.3$ is assumed
 529 (de Lima & Lovejoy, 2015; Nogueira, 2018). In Equation (11), the 2-year resolution time-
 530 series were assumed to have a constant value for every month inside each 2-years period.

531 Notice that a resolution limit should exist to the proposed stochastic downscaling
 532 algorithm, namely at time-scales below ~ 10 days where a fundamental transition occurs
 533 in the scaling behavior of most atmospheric fields (including global-averaged
 534 precipitation, see e.g. Lovejoy & Schertzer, 2013; Lovejoy, 2015; de Lima & Lovejoy,
 535 2015; Nogueira, 2017a,b, 2018). At faster time-scales intermittency becomes non-
 536 negligible and the quasi-Gaussian approximation to the statistics is no longer robust.

537 The proposed downscaling methodology corresponds to treating the sub-yearly
 538 frequencies as random ‘weather noise’, which is characterized, to a good approximation,
 539 by scale-invariant behavior with quasi-Gaussian statistics (Vallis, 2009; Lovejoy et al.,
 540 2015). A similar downscaling methodology has been previously demonstrated to
 541 reproduce the spatial sub-grid scale variability of topographic height (Bindlish & Barros,
 542 1996), precipitation (Bindlish & Barros, 2000; Reborá et al., 2006; Nogueira et al., 2013;
 543 Nogueira & Barros, 2015) and clouds (Nogueira & Barros, 2014).

544 Figure 5a showed that the PDFs computed from $\Delta P_{1m,DO}$, $\Delta P_{1m,Pr}$ and $\Delta P_{1m,SST}$ were in
 545 remarkable agreement with PDFs obtained from GPCP observational dataset for the
 546 1979-2010 period, representing a significant improvement compared to all atmospheric
 547 model-based products. This improved PDF accuracy was quantified using the Perkins
 548 skill score, S-Score (Perkins et al., 2007), defined as:

$$549 \text{ S-Score} = 100 \times \sum_{i=1}^M \min[f_{mod}(i), f_{obs}(i)] \quad (12)$$

550 where $f_{mod}(i)$ and $f_{obs}(i)$ are respectively the frequency of the modeled and observed P
 551 anomaly values in bin i , M is the number of bins used to compute the PDF (here $M=15$),
 552 and $\min[x,y]$ is the minimum between the two values. The S-Score is a measure of
 553 similarity between modeled and observed PDFs, such that if a model reproduces the
 554 observed PDF perfectly then $\text{S-Score}=100\%$.

555 The linear-based models showed S-Score values around 95%, which were significantly
 556 higher than the $\sim 80\%$ found for the atmospheric model-based products (Fig. 6).
 557 Furthermore, the stochastic model captured the change in the PDFs between two separate
 558 periods (1979-1990 and 1999-2010, Fig. 5b), while preserving the remarkably high
 559 ($\geq 90\%$) S-Scores (Fig. 6, blue and red markers). Indeed, the S-Scores for linear-based
 560 were consistently better than the S-Scores obtained from atmospheric model-based
 561 products ($\sim 80\%$). Despite some differences between PDFs obtained from $\Delta P_{1m,DO}$,
 562 $\Delta P_{1m,Pr}$ and $\Delta P_{1m,SST}$, their similar performance in reproducing observations was
 563 somewhat unexpected, given the distinct performances in reproducing the observed time-

564 series at multi-year resolutions. While the error analysis here was based on a limited
565 sample (mainly due to short duration of the satellite-period), these results suggested that
566 the proposed stochastic downscaling mechanism is quite robust in reproducing the
567 monthly statistics of **global-averaged precipitation**, with only moderate sensitivity to the
568 coarse resolution forcing.

569

570 **6. Discussion and Conclusions**

571 Atmospheric variables display significant variability over a wide range of temporal
572 scales, both due changes in external forcings (including surface fluxes, changes to
573 greenhouse gases and aerosol concentrations, solar forcing, and volcanic eruptions), but
574 also due to intrinsic modes of atmospheric variability. Additionally, external and internal
575 atmospheric processes interact nonlinearly amongst themselves, resulting in an intricate
576 multi-scale structure, which is still ill understood and responsible for significant
577 uncertainties in climate models. Here a multi-scale analysis framework was employed,
578 aiming to disentangle the complex structure of global-averaged precipitation variability.

579 **The multi-scale correlation structure obtained from Haar fluctuations suggested that**
580 **global-mean precipitation variability at multi-year time-scales is linearly related to the**
581 **net atmospheric radiative fluxes, corresponding to the dominant effect of energetic**
582 **constraints on precipitation variability. Furthermore, this linear relationship is dominated**
583 **by its longwave component and, more specifically, by the surface longwave radiative**
584 **fluxes, particularly DLR. The results also suggest that clouds play a negligible effect in**
585 **these linear correlations at multi-years scales.**

586 **Building on previous works of Lovejoy et al. (2017) and Nogueira (2018), the present**
587 **manuscript highlights a critical transition in the multi-scale correlation structure at time-**
588 **scales \sim 1-year, revealing a change in the control mechanisms of several atmospheric**
589 **variables, including precipitation. Specifically, at multi-year time-scales: (i) global-**
590 **averaged precipitation becomes tightly correlated to the net atmospheric radiative fluxes**
591 **($|\rho| \gtrsim 0.8$), and this correlation is dominated by the downwelling longwave radiative flux**
592 **at the surface; (ii) the cloud effects on the atmospheric radiative fluxes in (i) can be**
593 **neglected; (iii) global-averaged precipitable water vapor becomes tightly correlated**
594 **($\rho \sim 0.9$) to surface temperature. The respective sensitivity coefficient for multi-year**
595 **fluctuations of precipitable water vapor to surface temperature is estimated here to be**
596 **0.07%/K, close to the value predicted by the Clausius-Clapeyron relationship; (iv) global-**
597 **averaged SST and near-surface air temperature over land become strongly correlated**

598 ($\rho \sim 0.7$), implying a strong atmosphere-ocean coupling in agreement and extending the
599 results from Lovejoy et al. (2017) based on one observational dataset. In contrast, at sub-
600 yearly time-scales, the magnitude of all these correlations decreases abruptly towards
601 negligible values, and cloud effects are no longer negligible in the correlations between
602 atmospheric radiative fluxes and precipitation. Hints of a similar, but less robust,
603 transition also emerged for the correlation structure between global-averaged
604 precipitation and surface temperature - with negligible correlations at sub-yearly time-
605 scales, which tend increase at multi-year time-scales, although the latter displayed
606 significant spread amongst different datasets (ranging between ~ 0.4 to ~ 0.7).

607 The transition found here also contributes to sharpen the picture from previous studies
608 reporting a ‘slow’ response where global-averaged precipitation increases due to
609 increasing surface temperature, and a ‘fast’ response in which the atmosphere rapidly
610 adjusts to changes in top of atmosphere radiative forcing, and that is independent of
611 temperature fluctuations (Allen & Ingram, 2002; Bala et al., 2010; Andrews et al., 2010;
612 O’Gorman et al., 2012; Allan et al., 2014). The correlation structure found here suggests
613 that the ‘slow’ component corresponds to multi-year time-scales, and that radiative
614 constraints (particularly surface longwave fluxes) are the key mechanism controlling
615 precipitation variability rather than temperature, while cloud effects are negligible. On
616 the other hand, the correlations here confirm the break down of the linear relation between
617 temperature fluctuations at fast (sub-yearly) time-scales, but the dominant effect of top of
618 atmosphere radiative forcing is not evident and, most likely, the situation is much more
619 complex (for example surface sensible heat fluxes seem to become relevant at these time-
620 scales).

621 The robustness of this emergent multi-scale correlation structure is demonstrated by
622 proposing simple models for reconstruction of global-averaged at multi-year time-scales.
623 Anomaly time-series for global-averaged precipitation at 2-year resolution were derived
624 from SST observations alone, either directly based on precipitation vs SST correlation
625 structure, or by combining the more robust energetic constraints of global-averaged
626 precipitation (namely the precipitation vs clear-sky DLR correlation) with empirical
627 algorithm for clear-sky DLR estimation, and the Clausius-Clapeyron relationship. After
628 correcting for their systematic mean bias, the highly-idealized model for ΔP_{2y} based on
629 clear-sky DLR estimated from the Prata algorithm displayed similar accuracy in
630 reproducing observations as atmospheric model-based products, as measured by RMSE_{bc},
631 Pearson correlation coefficient and normalized standard deviation. Finally, the model

632 based on precipitation vs SST correlation showed the weakest performance, which agrees
633 with the less robust correlations underlying this idealized model.

634 The proposed linear models cannot be extended to sub-yearly the time-scales because all
635 the correlations upon which they rely become negligible. This abrupt transition in the
636 multi-scale correlation structure implies that at sub-yearly time-scales the tight linear
637 coupling between atmospheric and ocean temperature, the Clausius-Clapeyron
638 relationship, and the atmospheric energy balance are no longer dominant linear
639 constraints for global-averaged. Nonetheless, the multi-scale analysis framework
640 provides another path for reconstruction of the precipitation statistics at sub-yearly
641 resolution. A stochastic downscaling algorithm based on scale-invariant symmetries of
642 precipitation was applied to ΔP_{2y} reconstructed time-series, resulting in monthly global-
643 averaged precipitation PDFs. Remarkably, the reconstructed PDFs at monthly resolution
644 showed better accuracy in reproducing observed statistics than atmospheric model-based
645 products, as measured by the PDF matching S-Score computed over decadal and 30-year
646 periods. These results highlight the relevance and potential applications of multi-scale
647 frameworks for atmospheric sciences.

648

649

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665 https://eosweb.larc.nasa.gov/project/ceres/ebaf_surface_table

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806 **Table 1** Linear regression coefficient $\alpha_{W,SST}$ estimated from $\Delta PWV/PWV_c$ against ΔSST
807 at 2-year resolution, assuming a relationship as given by Equation (1). The respective
808 coefficient of determination is also provided. The $\alpha_{W,SST}$ are computed for ERA-20C,
809 20CR, and for the ensemble of ERA-20CM simulations. Additionally, the coefficient is
810 estimated by pooling together ERA-20C, ERA-20CM (ensemble) and 20CR datasets.

Dataset	$\alpha_{PWV,SST} [K^{-1}]$	R^2
ERA-20C	0.09	0.97
20CR	0.10	0.92
E20CM (Ensemble)	0.07	0.92
All Datasets	0.08	0.91

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813 **Table 2.** Linear regression coefficient $\alpha_{P,DLR}$ estimated from $\Delta P/P_c$ against ΔDLR at 2-
814 year resolution, assuming a relationship as given by Equation (11). The respective
815 coefficients of determination are also provided. The $\alpha_{P,DLR}$ values are computed for ERA-
816 20C, 20CR, and for the ensemble of ERA-20CM simulations. Additionally, the
817 coefficient is estimated by pooling together all datasets, including GPCP observations
818 against DLR from CERES-EBAF.

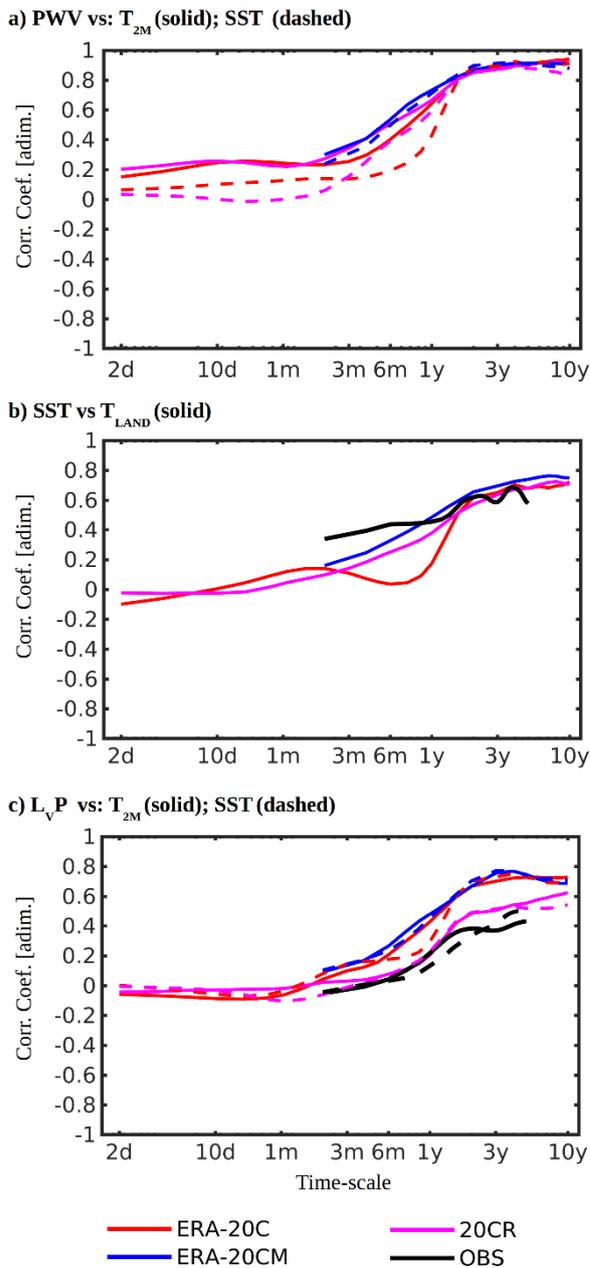
Dataset	$\alpha_{P,DLR} [(Wm^{-2})^{-1}]$	R^2
ERA-20C	0.005	0.88
20CR	0.003	0.51
E20CM (Ensemble)	0.004	0.81
All datasets (includes observations)	0.004	0.70

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821 **Table 3.** Linear regression coefficient $\alpha_{P,SST}$ estimated from $\Delta P/P_c$ against ΔSST at 2-
822 year resolution. The respective coefficients of determination are also provided. The $\alpha_{P,SST}$
823 values are computed for ERA-20C, 20CR, for the ensemble of ERA-20CM simulations,
824 and for GPCP against ERA-20CM control SST forcing. Additionally, the coefficient is
825 estimated by pooling together all datasets.

Dataset	$\alpha_{P,SST} [K^{-1}]$	R^2
ERA-20C	0.04	0.89
20CR	0.02	0.35
E20CM (Ensemble)	0.02	0.73
GPCP	0.04	0.42
All datasets (includes observations)	0.02	0.53



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827 **Figure 1.** Cross-correlation coefficients against temporal scale computed from Haar
 828 fluctuations for global-mean time-series of a) PWV vs T_{2m} (solid) and PWV vs SST
 829 (dashed); b) SST vs T_{land} ; and c) L_vP vs T_{2m} (solid) and L_vP vs SST (dashed). Red lines
 830 represent results from ERA-20C, blue lines are from ERA-20CM, pink lines are from
 831 20CR and black lines are estimated from observational products.

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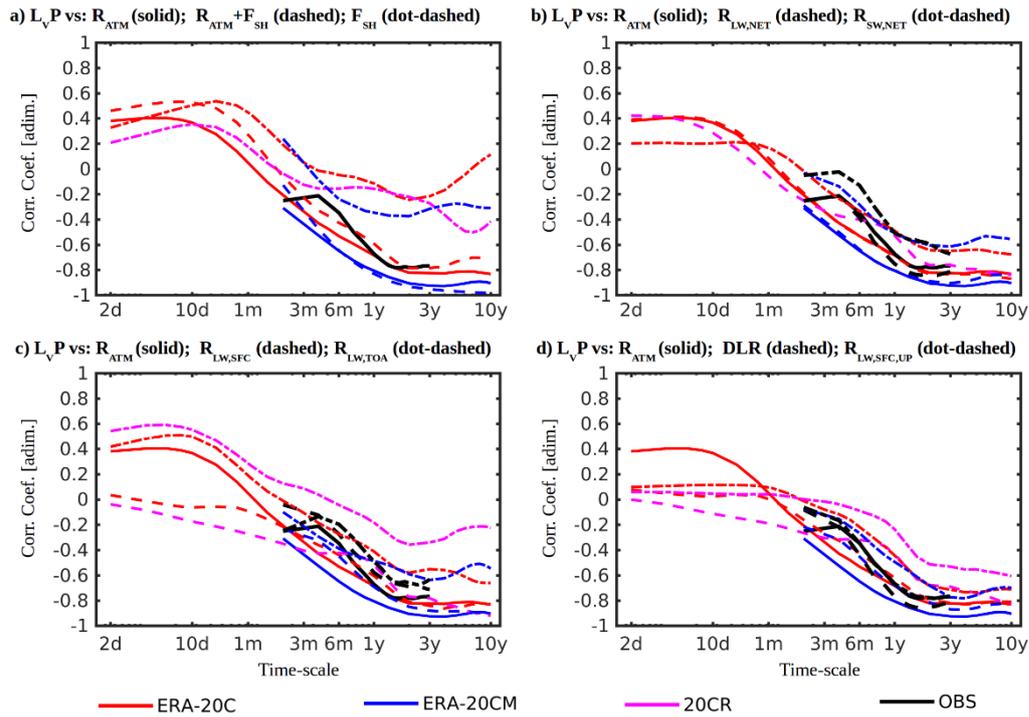
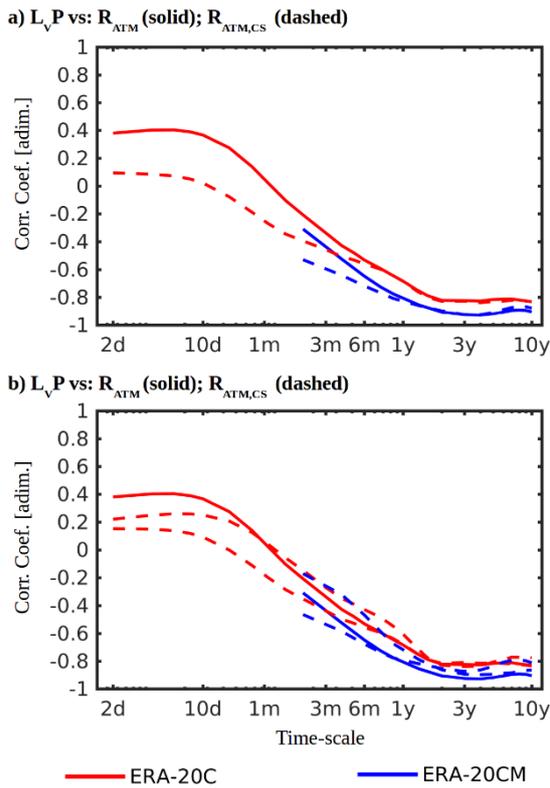


Figure 2. Cross-correlation coefficients against temporal scale computed from Haar fluctuations of a) $L_v P$ vs R_{atm} (solid), $L_v P$ vs $(R_{atm} + F_{SH})$ (dashed) and $L_v P$ vs F_{SH} (dot-dashed); b) $L_v P$ vs R_{atm} (solid), $L_v P$ vs $R_{LW,net}$ (dashed), and $L_v P$ vs $R_{SW,net}$ (dot-dashed); c) $L_v P$ vs R_{atm} (solid), $L_v P$ vs $R_{LW,SFC}$ (dashed), and $L_v P$ vs $R_{LW,TOA}$ (dot-dashed); and d) $L_v P$ vs R_{atm} (solid), $L_v P$ vs DLR (dashed), and $L_v P$ vs $R_{LW,SFC,UP}$ (dot-dashed). Red lines are computed from ERA-20C, blue lines are from ERA-20CM, pink lines are from 20CR and black lines are computed from GPCP and CERES-EBAF observational products. Notice that R_{atm} and $R_{SW,net}$ weren't available from 20CR, while sensible heat flux wasn't available from observations.



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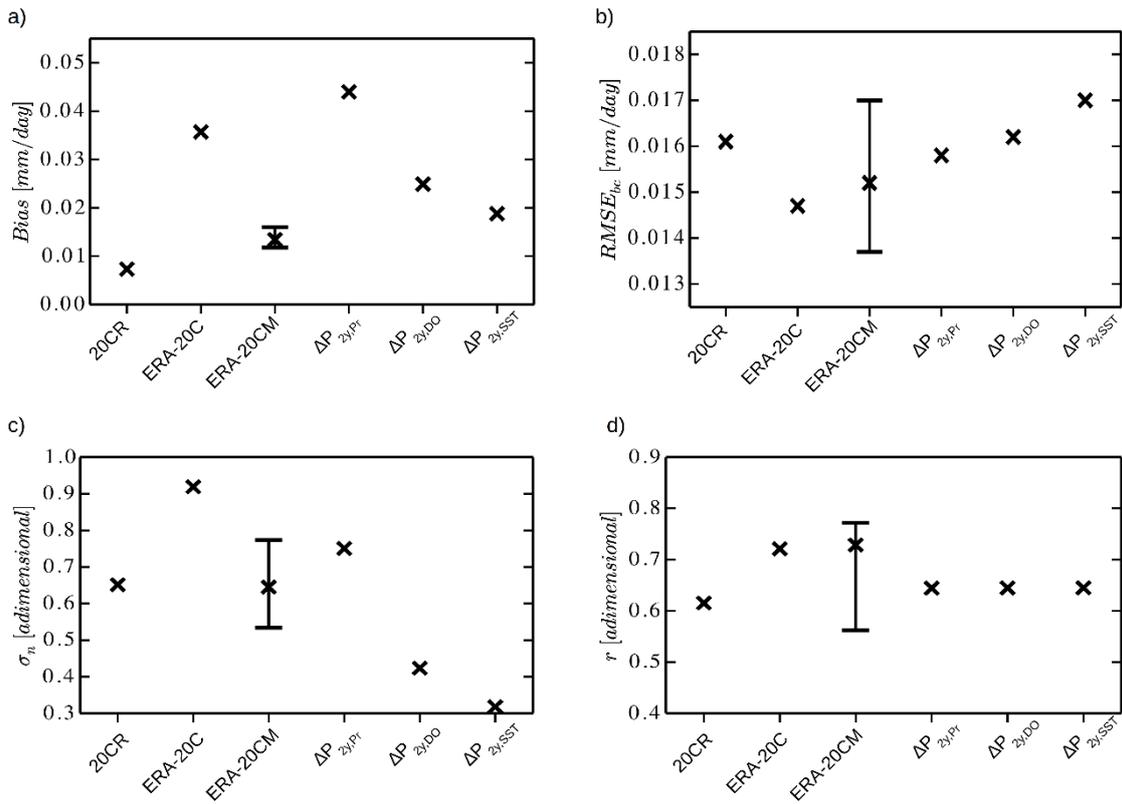
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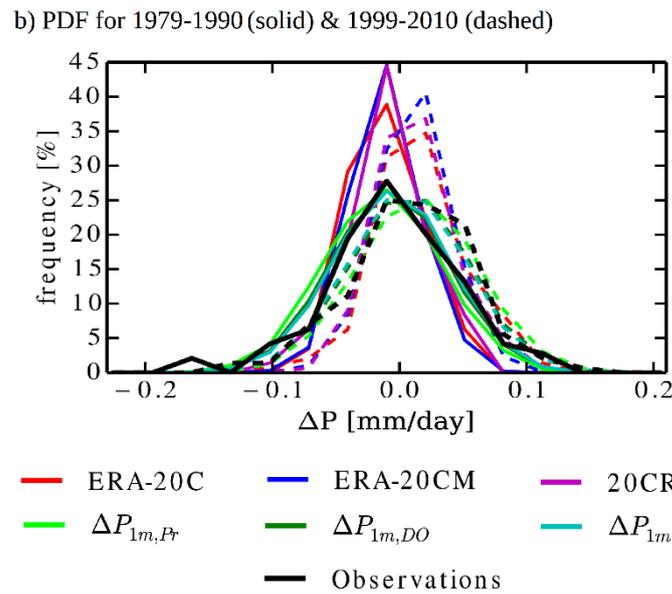
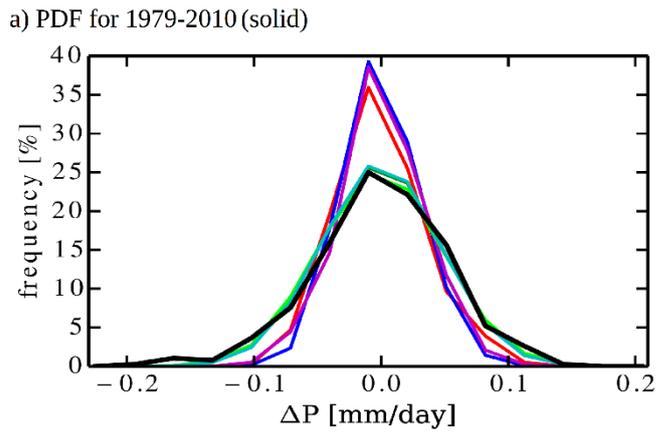
Figure 3. Cross-correlation coefficients against temporal scale computed from Haar fluctuations of a) $L_v P$ vs R_{atm} (solid) and $L_v P$ vs $R_{atm,CS}$ (dashed); b) $L_v P$ vs $R_{LW,SFC}$ (solid) and $L_v P$ vs $R_{LW,SFC,CS}$ (dashed). Red lines are computed from ERA-20C and blue lines are from ERA-20CM.



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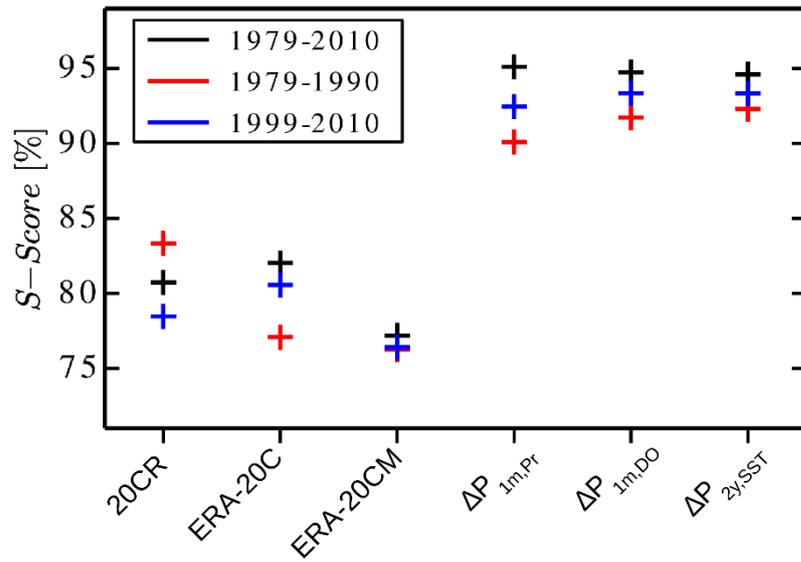
853 **Figure 4.** Error estimates from simulated anomaly time-series for P at 2-year resolution
 854 against GPCP, computed for the 1979-2010 period, including a) mean bias (Bias); b) root-
 855 mean-square error after bias correction (RMSE_{bc}); c) model standard deviation
 856 normalized by observed standard deviation (σ_n); and d) Pearson correlation coefficient
 857 (r). For ERA-20CM dataset the range for all ensemble members is shown, while ‘x’ marks
 858 their mean value. The p-value for all correlations shown in panel (d) are <0.05.

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Figure 5. PDFs estimated from monthly anomaly time-series for P from ERA-20C (red), ERA-20CM (dark blue), 20CR (pink), GPCP (black), $\Delta P_{1m,DO}$ (dark green), $\Delta P_{1m,Pr}$ (light green), and $\Delta P_{1m,SST}$ (light blue). In panel a) the PDFs are estimated for the 1979-2010 period, and in panel b) the PDFs are estimated for the 1979-1990 period (solid) and the 1999-2010 period (dashed).



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869 **Figure 6.** S-Score computed from the different P simulations against GPCP. The values
 870 estimated for the full satellite period (1979-2010) are presented in black, for the 1979-
 871 1990 period are presented in red, and for 1990-2010 period are presented in blue. For
 872 ERA-20CM dataset, the S-Score is estimated from the 10-member ensemble PDF.