The author presents an idea for dealing with the low likelihood high impact probability portion of future climate projections. These, we assume, would form storylines or narratives of future global or regional climate. The rather simple, or even naïve, premise is that likelihood would equal the impact times the risk (R=LI). We use the author's own graphic (reverse engineered) as a starting point and show that additional risk curves have multiple roots (one, two or even three x-axis values for a given y-axis value). We also know from this detailed analytical analysis, that the risk curves are highly sensitive to any assumed exponential (or even linear) impact curve. Finally, the asymptotic behavior of the assumed probability function (PDF), or its corresponding cumulative probability function (CDF), doea not comport well with an assumed exponential impact (risk equals zero for Equilibrium Climate Sensitivity (ECS) >> 1 or risk equals infinity for ECS >> 1).

We first determined the author's chosen distribution, impact and resulting risk curves using WebPlotDigitizer. We found that the distribution is virtually certain (IPCC AR5 WGI uncertainty language) to be a two-parameter gamma distribution (alpha and beta coefficients) as shown in Figure 1 (all yellow colored curves are reproductions of the author's chosen curves).

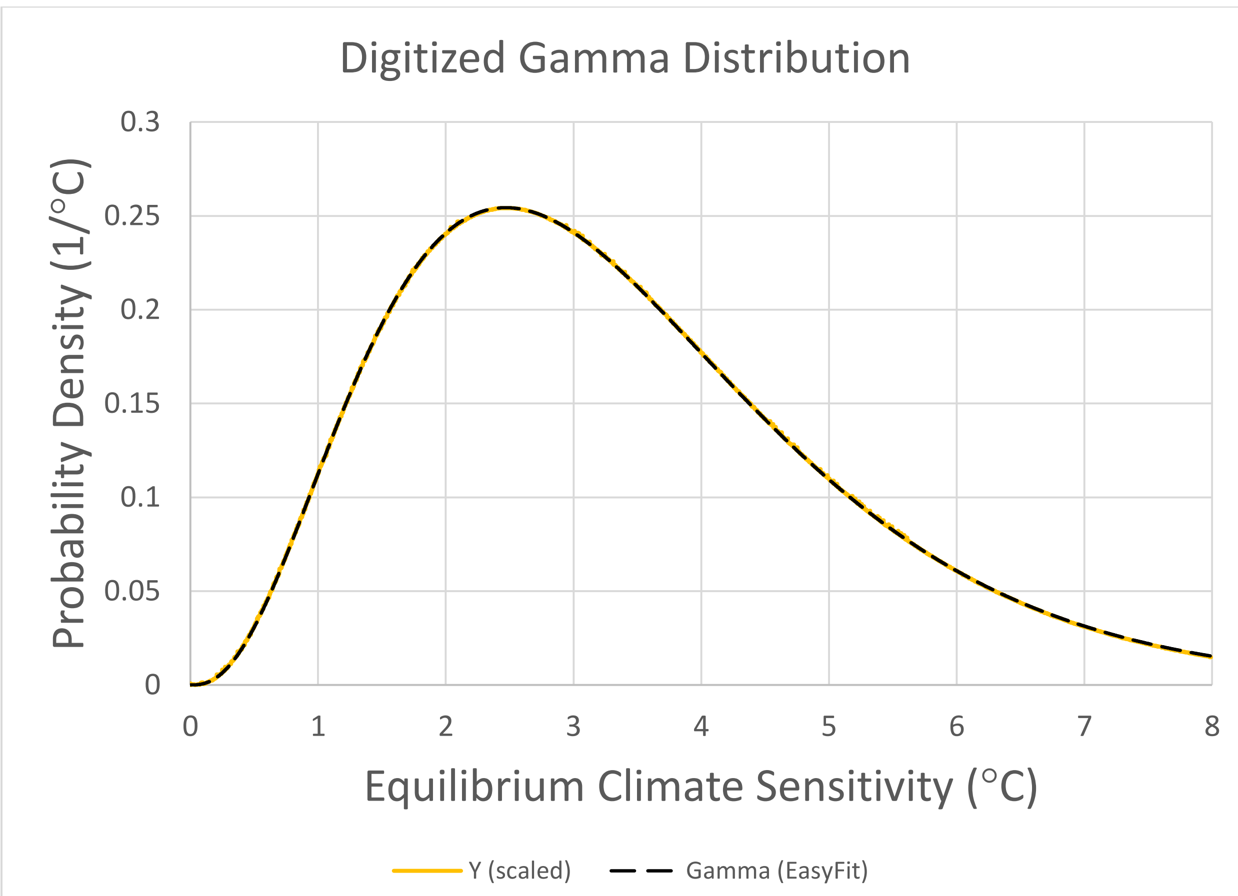

Figure 1. Author's sample PDF (note units and precise y-axis via unitary nature of PDF's).

We then determined, again with virtual certainty, that the author's impact curve was exponential in nature and that at X = 0 that Y = 0 as shown in Figure 2 (the curve starts with a finite slope due to the nature of exponentials, in that the y-axis asymptote by definition only occurs at minus infinity). The units shown reflect the digitized y-axes where a value of zero (bottom horizontal line) to one (top horizontal line) was assumed at the start for all three digitized curves

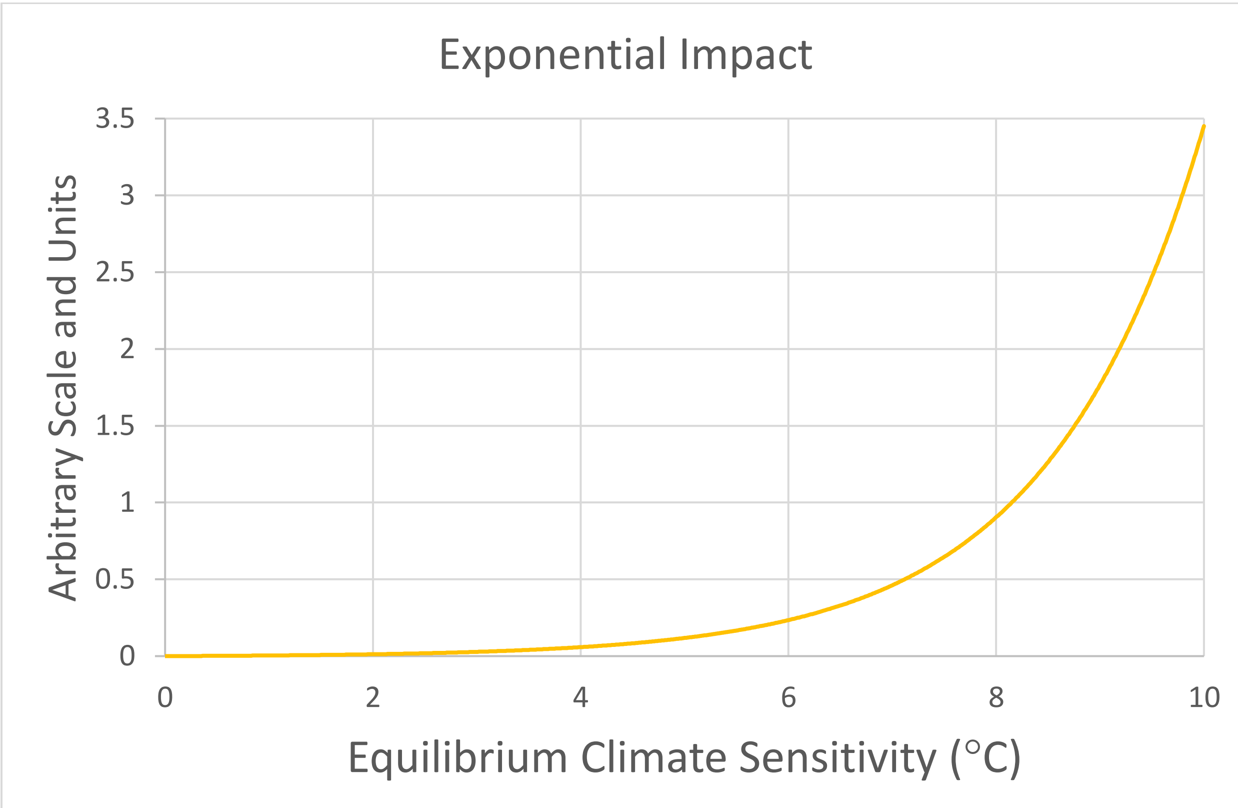

Figure 2. Author's sample exponential impact curve (note y-axis arbitrary scale and units).

The author's risk curve was reproduced via an optimization process (to determine an initial arbitrary y-axis offset for the impact curve) of the digitized impact and risk curves (the digitized gamma curve was assumed to be correct). This is, or should be, a routine procedure and is beyond the scope of this specific note. We know two properties of the author's three curves via; (a) the unitary nature of any PDF (unit area under the PDF curve, can assign units to the y-axis) and (b) using the L=R/I relationship the y-axis scale ratio between R and I can be determined. The y-axis scale ratio was found to be, I/R = 62.6. Given L and given I/R we assume any arbitrary scale and units for I. Figures 3 and 4 shows the original version of the author's risk curve (I/R relationship not applied) after optimization for maximum R^2 of the risk regression relationship.

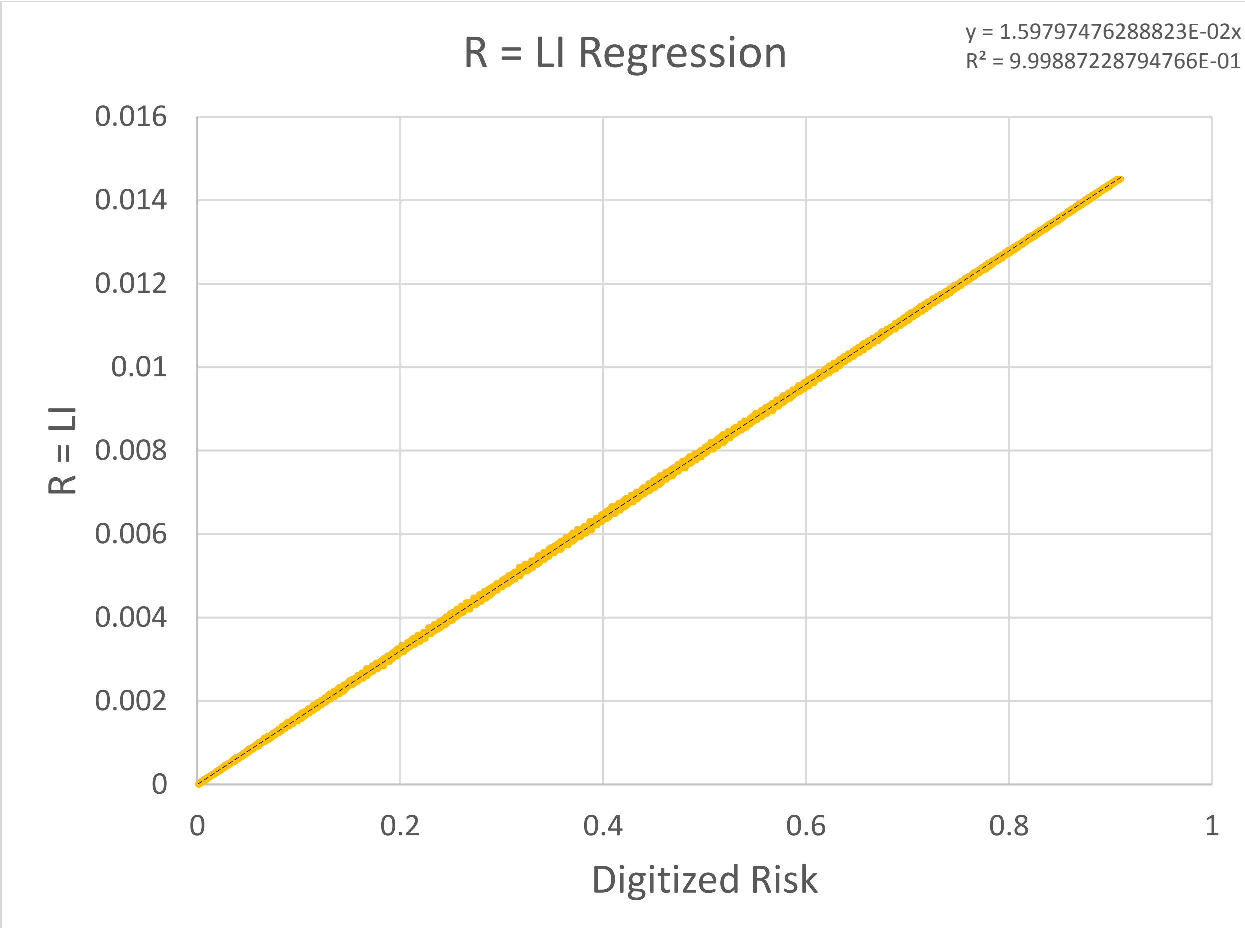

Figure 3. Risk regression fit using author's digitized risk curve.

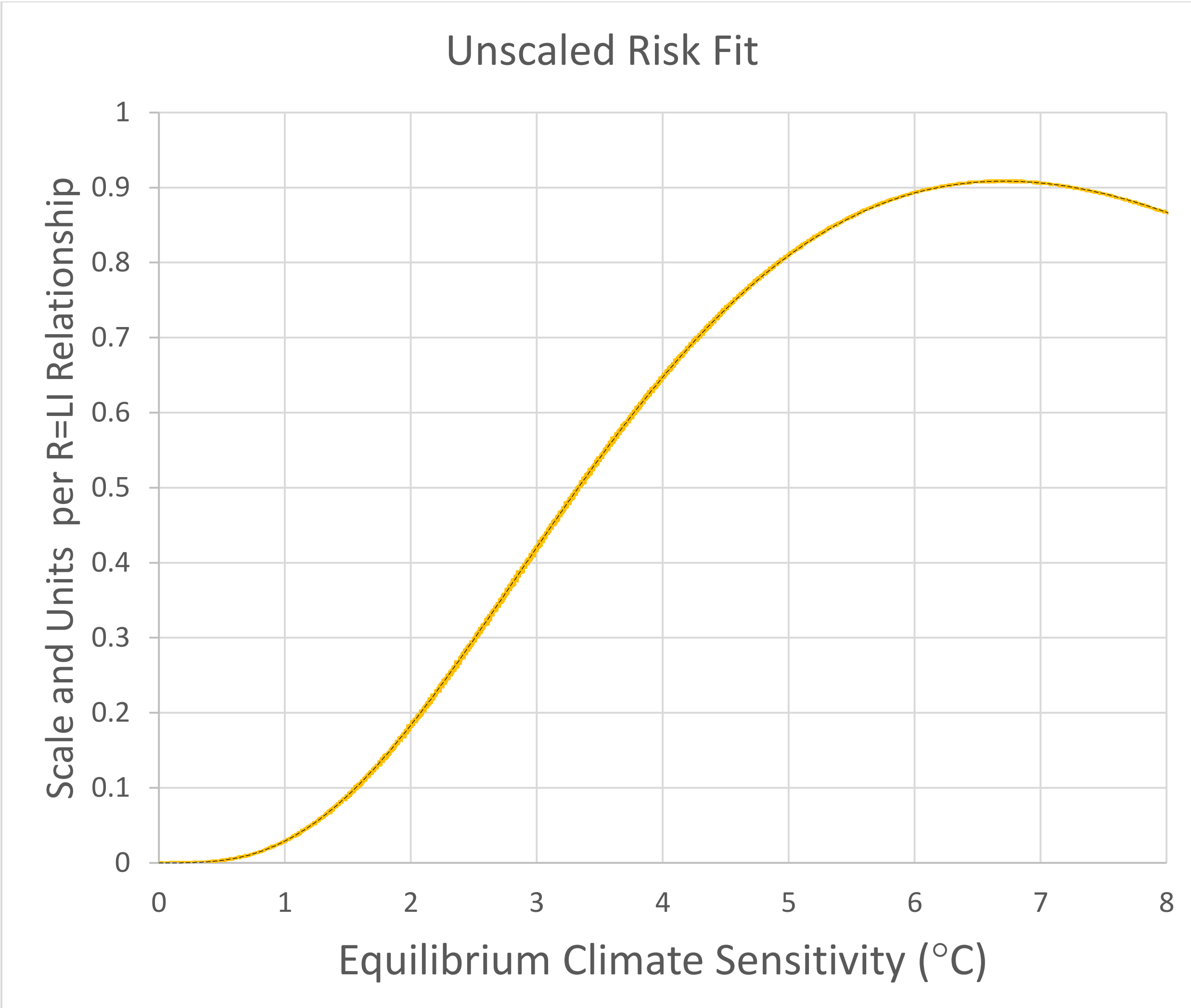

Figure 4. Author's risk curve with original digitized unitary y-axis scale.

We decided to add additional PDF's as the author's risk curve suggests two sets of x-axis values for ECS (the downturn seen in Figure 4) for any and/or all y-axis values. We used a commercial-of-the-shelf (COTS) program and chose the following distributions; (i) gamma, (ii) log-logistic, (iii) inverse gaussian, (iv) lognormal and (v) fatigue life. All five distributions have three parameters, where the third is a location parameter (starting point of one-sided distributions). We also chose three sets of ECS based on the IPCC AR5 WGI conditionals;

(a) IF(ECS.LE.1)THEN(P.LE.0.05)
(b) IF(ECS.GE.1.5.AND.ECS.LE.4.5)THEN(P.GE.0.66.AND.P.LE.1.00)
(c) IF(ECS.GE.6)THEN(P.GE.0.9)

where the upper-case P is taken as the CDF. Figures 5 and 6 show the results of the selection and design process of the PDF's.

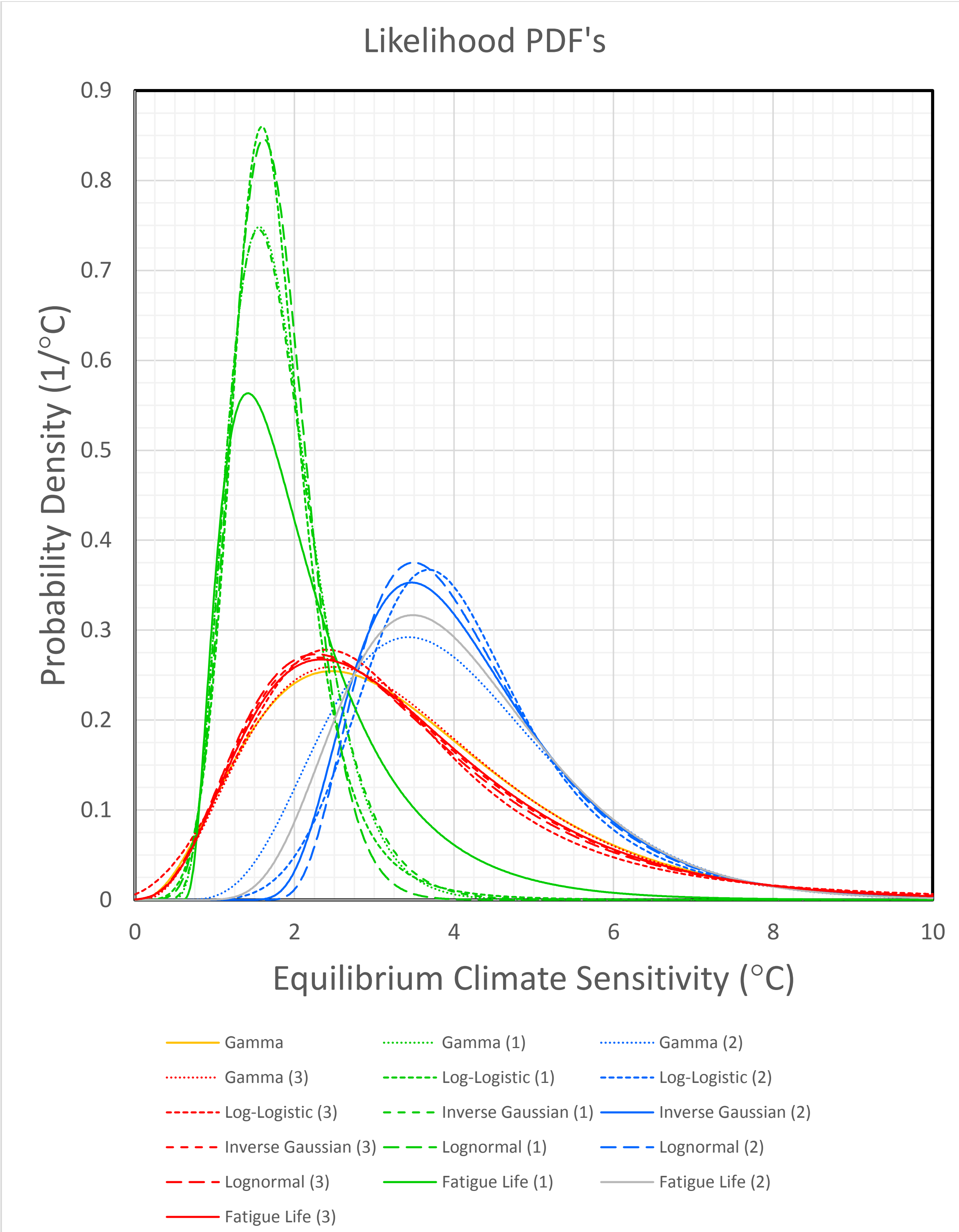

Figure 5. PDF's for low (green), medium (red) and high (blue) categories bounded via the three IPCC AR5 WGI ECS conditional statements. Author's PDF curve shown in yellow.

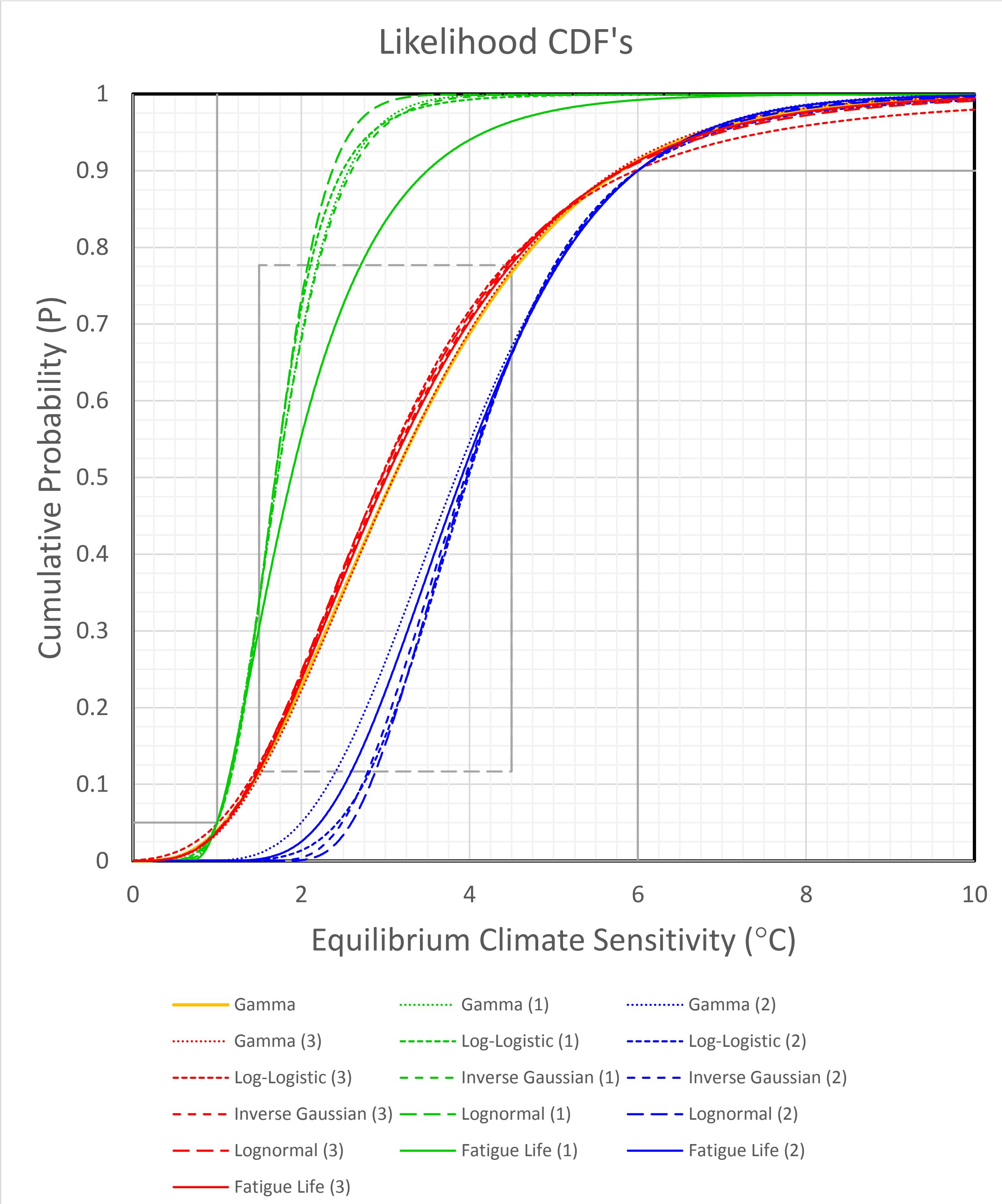

Figure 6. CDF's for low (green), medium (red) and high (blue) categories bounded via the three IPCC AR5 WGI ECS conditional statements. Author's CDF shown as a solid yellow line. Interior grey lines are bounding IPCC conditionals. The lower left corner of the P = 0.66 inner box is pinned to the author's sample distribution (author's distribution just missed the second conditional, P=0.65, but is still quite close to our medium (red) gamma distribution, P = 0.66).

Figure 6 was set up so that the low ECS (green) lines met exactly conditionals (a) and (b) while P = 1 (roughly speaking) at an ECS = 6 degree centigrade. Conversely, the high ECS (blue) lines in Figure 6 met exactly conditionals (b) and (c) while P = 0 (roughly speaking) at an ECS = 1 degree centigrade. We note that these are not unique single valued lines and that they do not span the entire range of possible IPCC conditional criteria. Finally, the medium (red)) lines in Figure 6 were intended to meet all three conditionals, but none of the five distributions were able to meet all three conditionals at the same time. We decided that the minima of the root mean square error (RMSE) provided the closest fit and would roughly balance out conditionals (a) and (c), it turned out that all five medium (red) lines exactly met conditional (b) P = 0.66. We find the current IPCC conditionals to be quite flexible, if fact too flexible, as the entire CDF can be completely constrained between 1.5 and 4.5 degree centigrade (P = 1.0), we can select both low and high PDF's that quite literally approach either the the 1.5 or the 4.5 degree centigrade vertical lines or a nearly vertical line anywhere in between 1.5 and 4.5 (narrow banded but still have a heavy or fat tailed positive skewed distribution).

With the likelihood defined by the 15 PDF's (or 16 including the author's curve) from Figures 5 and/or 6 and using the author's impact curve as shown in Figure 2 we arrive at the risk curves as shown in Figure 7 via the simplistic R = LI relationship.

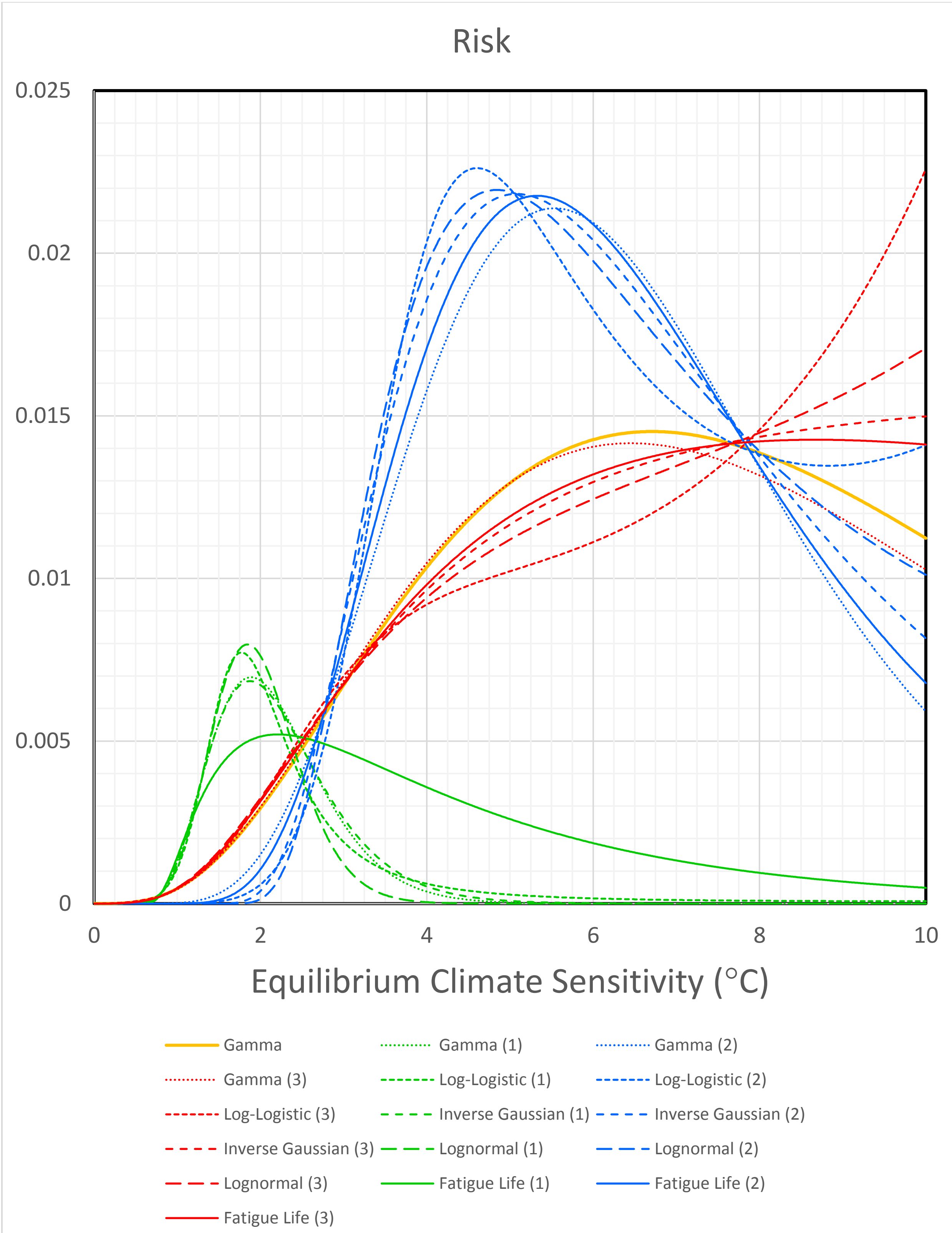

Figure 7. Risk curves for low (green), medium (red) and high (blue) ECS categories bounded via the three IPCC AR5 WGI ECS conditional statements. Author's risk curve shown as a solid yellow line.

We conclude from Figure 7 that risk is ill defined as there exists multiple rooted x-axis values of ECS for most of these risk curves and that the exponential nature of the impact curve yields strange asymptotic behaviors (not shown in Figure 7 as these appear beyond the right-hand side of Figure 7). As with Figure 6, we find that the risk domain can be (more or less) fully occupied regardless of the underlying impact curve assumed (excepting large numerical values for the exponential coefficient itself). Furthermore, we find a linear impact curve will result in high risk values for the low (green) ECS and high (blue) ECS curves while the medium (red) ECS curves occupy a valley of lower risk values between the low and high ECS curves. Figure 8 shows this behavior using the lowest 1% of the exponential impact curve (where the exponential curve is linear itself).

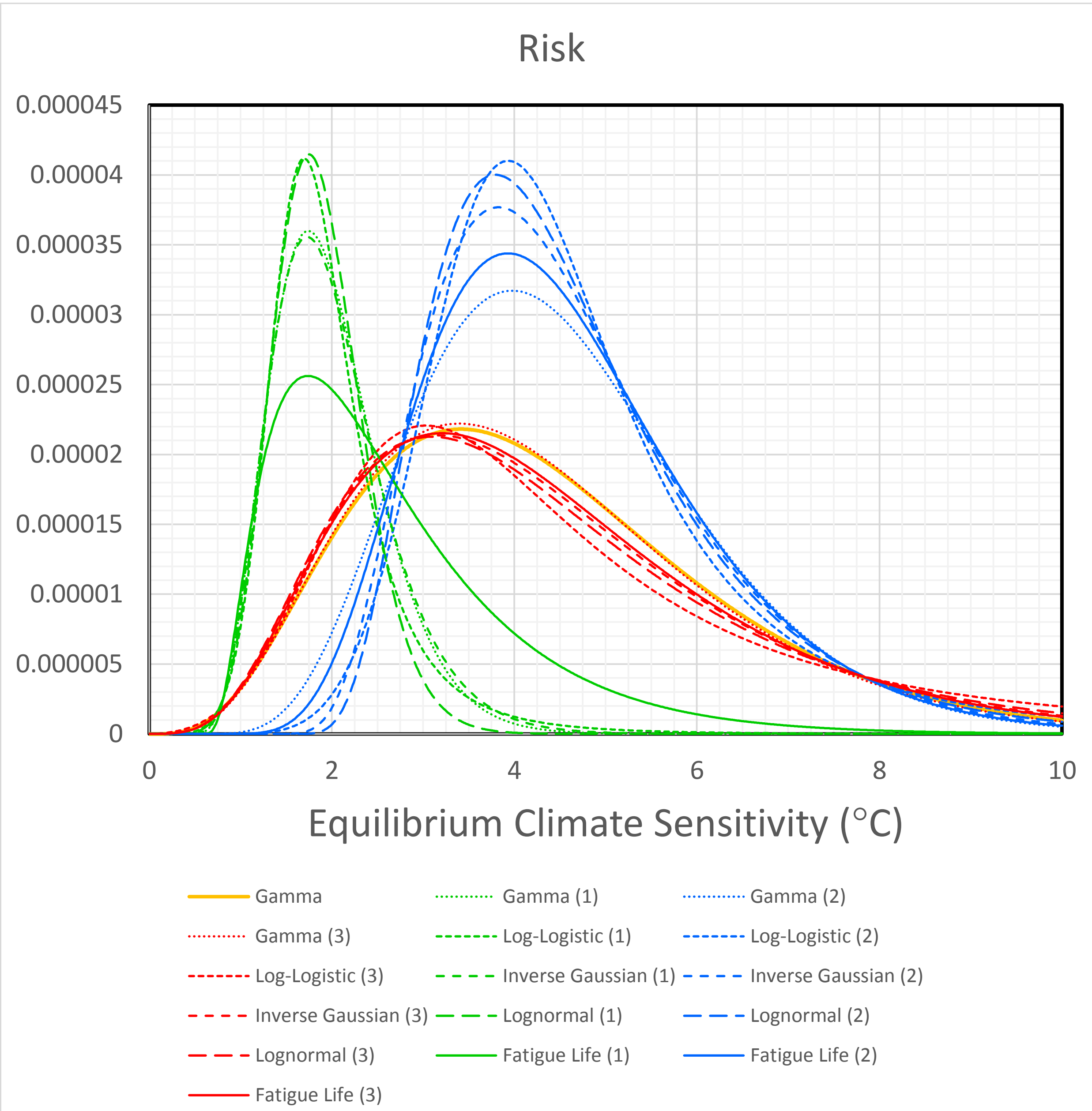

Figure 8. As in Figure 7 but with a linear impact relationship. Note that the y-axis is arbitrary.

We come to at least one firm conclusion, that the IPCC AR6 WGI is ill prepared to handle such gross ambiguities in either its further narrowing of ECS guidance (however narrow their ECS conditionals become they still will leave much room for PDF selections) or in assuming a class of impact curves (which we believe have significant uncertainties themselves, perhaps as large as the first significant itself).

If one were to do so, we would recommend that the author use a quasi-monotonic relationship in place of the ECS probability curves. Such examples would be projected global mean surface temperature (GMST) or projected ocean heat content (OHC) or projected global mean sea level (GMSL). Further, although we have seen this particular approach in practice (using the PDF in its entirety), we are still always concerned with the heavy or fat tailed behaviors itself (e. g. extreme value theory). While it is all well and good to run millions of joint probability PDF's through Monte Carlo simulations (which themselves would revert to something similar to a symmetric normal distribution, or so we have been told) we see no value to this methodology when the overriding conditional is a plausible and somewhat accurate rendition of the fat or heave tailed behavior itself (which is definitely not normally distributed). Exponential growth (impact) multiplied by asymptotic exponential-like decay behaviors (PDF's) and a so-called "hump" right in the middle of said PDF's is bound to produce weird multi rooted behaviors in an overly simplistic R = LI relationship.

We would appreciate it, if the author is willing, to see a fuller description of risk and its applications to low likelihood high impact storylines, narratives or scenarios as currently envisioned by IPCC AR6 WGI. In other words, this appears to be the time and place to present a full paper for peer review that further explains and flushes out this idea that the author has proposed.

An Excel spreadsheet is provided from whence all eight figures were derived (or extracted). We also note that it was our original intent to go it alone (without any direct or indirect input from the author at any point in this review process itself, such that any errors, misinterpretations or misrepresentations are our own and are not, in any way, shape or form, the original author's fault).