We would like to thank referee #1 for the discussion. Below we reply to the comments in detail.

1) About the maximum and minimum. The theory in this paper follows the principle of Maximum Entropy Production which states that certain complex non-equilibrium thermodynamic systems can be successfully characterised as being in states in which the rate of thermodynamic entropy production is maximised (e.g., Kleidon, 2016). Mixing processes in estuaries continuously perform work by depleting salinity gradients. In doing so, these processes produce entropy, following the natural direction given by the second law of thermodynamics. So we assume the power to mix is maximized to maximize the entropy production in this system.

2) About the derivation.

The parameters along an estuary (i.e. $A$, $D$, and $S$) in this research are all tidal-averaged.

The relations between the parameters according to the salt balance equation, \( \frac{\partial S}{\partial t} = \frac{\partial}{\partial x} \left( QS - DA \frac{\partial S}{\partial x} \right) \), can be considered as (for deriving $S'$):

\[
(S - S_f) \text{ was simplified by } S.
\]

However, when the optimum situation is achieved, it is a steady state and $S' = QS/AD$.

For calculating the parameters along the estuary in optimum situation, all the parameters are functions of $x$ only (thick line route in the above scheme):

\[
\frac{dS'}{dD} = \frac{dS'}{dx} \frac{dx}{dD} = \left( \frac{\partial S'}{\partial S} \frac{\partial S}{\partial x} + \frac{\partial S'}{\partial D} \frac{\partial D}{\partial x} + \frac{\partial S'}{\partial A} \frac{\partial A}{\partial x} \right) \frac{dx}{dD} = \left( \frac{Q}{AD} \frac{\partial S}{\partial x} - \frac{QS}{AD} \frac{\partial D}{\partial x} - \frac{QS}{AD} \frac{\partial A}{\partial x} \right) \frac{dx}{dD} = \frac{Q}{AD} \left( \frac{dS}{dx} \frac{dD}{dx} - \frac{S}{D} - \frac{S}{A} \frac{dA}{dx} \right) = \frac{Q}{AD} \left( \frac{S'}{D'} - \frac{S}{D} - \frac{S}{A} \frac{A'}{D'} \right) = 0
\]

where: $A' = \frac{dA}{dx}$, $D' = \frac{dD}{dx}$ and $S' = \frac{dS}{dx}$.

(3) About the sloping or flat bed. In fact, the slope is small so the increase of depth due to the salinity difference ($\Delta h$) is trivial compared to the depth at the estuarine mouth ($h$) (but $\Delta h$ is important for the mixing).

If, however, a downward slope were introduced in the picture, then we would have to include the bottom pressure as well: the water pressure over the additional depth near the downstream boundary would have to be balanced by the horizontal component of the pressure exercised on the water by the estuary bottom. This would make the sketch unnecessary complex.