

## ***Interactive comment on “Fractional Governing Equations of Transient Groundwater Flow in Confined Aquifers with Multi-Fractional Dimensions in Fractional Time” by M. Levent Kavvas et al.***

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### RESPONSE TO THE COMMENTS OF ANONYMOUS REFEREE #1

The authors thank the anonymous Referee #1 for the valuable comments.

Referee #1 General Comment:

"This study developed a governing equation of transient groundwater flow in a multi-fractional, multi-dimensional confined aquifer in fractional time. This study has suffi-

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cient novelty, and the developed governing equations would be important for groundwater modeling. Furthermore, this paper is documented well. However, the application and discussion parts of this paper are short. If the authors can provide more detailed information on the application, and more discussions on the results, it would be helpful for readers to understand importance of this study. "

Response: The authors thank the positive comments of the Referee #1.

Referee #1 Specific comments:

1. "Mathematical symbols such as time "t" and function "f" should be written in italic even in sentences. "

Response: These changes will be made in the revised manuscript as suggested.

2. "It would be helpful to readers if the authors provide more explanation why nonlocal governing equations can account for the influence of the initial and boundary conditions on the flow process more efficiently than the corresponding local-scale equations, in P.9 L.4-6. "

Response: In P.8 L.24-P.9 L.5, the authors explained the reason why Caputo derivative can better quantify the effect of the initial and boundary conditions than the conventional derivative. As discussed in P.8 L.24, the Caputo derivative is a nonlocal quantity. The fractional governing equations based on Caputo derivative are nonlocal governing equations that can more efficiently account for the influence of the initial and boundary conditions on the flow process more efficiently than the corresponding local-scale equations. Please see also the discussion on the physical framework of fractional governing equations of soil water flow in Kavvas et al. 2016. Following the recommendation of Referee #1, more explanation will be added in the revised manuscript.

3. "I highly recommend the authors to provide more detailed information of the setting of the numerical application in Section 6. It is possible to understand the numerical application setting if one reads Wang and Anderson (1995). However, it is currently

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difficult to know it from the description of this paper."

Response: The numerical example is based on the example provided in Wang and Anderson (1995). Detailed information of the numerical example used in this study was provided in P.9 L.13-16 and Figure 2: "In this example, groundwater flow is simplified to be one-dimensional. The length of the confined aquifer is 100 m. The aquifer has a hydraulic transmissivity (T) of 0.02 m<sup>2</sup>/min and a specific storage (S) of 0.002. The groundwater hydraulic head is initially uniform at 20 m. The water level downstream suddenly drops to 10 m and stays at 10 m. The total simulation time is 600 minutes." Given these information, we then solve the fractional governing equations when the orders of the fractional derivatives are 1, 0.9 and 0.8. Following the recommendation of Referee #1, more explanation will be added in the revised manuscript.

4. "In P.10 L.8-10, "the recession rate of the non-dimensional hydraulic heads from the initial condition also gets slower with respect to the case of the conventional governing equation with integer derivative powers." Please explain why this result is important on groundwater modeling."

Response: The modeling results may help explain the long-range dependence characteristics in some groundwater level datasets. The results may also shed light on the non-Fickian transport phenomena in groundwater flow. Explanation/implication of the modeling results will be added in the revised manuscript. A numerical solution methodology and additional numerical examples by the proposed fractional governing equations will be provided in Tu et al. (2017).

5. "I recommend the authors to add a description about the importance of the result of the numerical application at the end of Conclusion."

Response: Following the recommendation of Referee #1, a description about the importance/implication of the numerical application results will be added at the end of the conclusion section in the revised manuscript.

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#### References:

Kavvas, M. L., Ercan, A., and Polsinelli, J.: Governing equations of transient soil water flow and soil water flux in multi-dimensional fractional anisotropic media and fractional time, *Hydrol. Earth Syst. Sci.*, 21, 1547-1557, <https://doi.org/10.5194/hess-21-1547-2017>, 2017.

Tu, T., Ercan, A., and Kavvas, M. L.: Time-Space Fractional Governing Equations of Transient Groundwater Flow in Confined Aquifers: Numerical Investigation, *Hydrological Processes*, under review, 2017.

Wang, H. F. and Anderson, M. P.: *Introduction to groundwater modeling: finite difference and finite element methods*, Academic Press, 1995.

#### RESPONSE TO THE COMMENTS OF ANONYMOUS REFEREE #2

The authors appreciate the valuable general comments of anonymous Referee #2, and thank him/her for these comments. The authors' responses to the specific comments of the referee are provided below:

" a) I recommend further elaboration on the implementation and discussion of the theoretical advances introduced in the manuscript - particularly in the context of the broader Geo Sciences. That way, the interdisciplinary readership of the journal can better appreciate the developments and findings."

Response: Following the recommendation of Referee #2, the authors will add material to the revised manuscript that will further attempt to elaborate and discuss the theoretical work in the paper in the context of broader Geosciences.

" b) Similarly to what had been noted by the other reviewer, scalar variables or functions should be typeset in italic even in the main body of the text."

Response: Following the recommendation of Referee #2, the scalar variables or functions will be typeset in italic in the revised manuscript.

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